Non-exponential dynamics near the continuum threshold: Bound states, anti-bound states and exceptional points

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Exponential decay is ubiquitous in nature. Well-known quantum examples include nuclear decay and the typical relaxation dynamics of atoms or molecules. It is less widely known that deviations from exponential decay occur at very short and extremely long timescales in virtually all such scenarios. These deviations appear due to the existence of a lower bound on the energy continuum associated with the environment into which the system decays. This lower bound can be understood as a branchpoint in the complex energy plane, which we refer to as the *continuum threshold*. Examples include the ionization energy of atoms or molecules or the band edges in a condensed matter system (in the latter case, the continuum typically has both lower and upper thresholds). Note that these non-exponential dynamics are examples of non-Markovian decay, or reversible dynamics that may be useful in quantum information processing; however, they are usually difficult to observe in experiment.

In this lecture, we will study various means by which the long-time deviations can be modified or enhanced, as well as other situations in which the threshold can influence dynamical processes in open quantum systems. For example, when a generalized eigenstate with real energy (bound state or anti-bound state) approaches the threshold, the long-time deviations can be divided into two time zones, with the timescale separating these two behaviors being inversely proportional to the energy gap between the energy eigenvalue and the threshold. While this yields fractional or incomplete decay in the case of a bound state, a delocalized anti-bound state near the threshold results in complete non-Markovian decay [**1**].

Further, we consider the influence of exceptional points (EPs), which are branchpoints in the parameter space of a given Hamiltonian at which two or more eigenstates coalesce into a single state. An EP can influence the spectral and dynamical properties of open quantum systems under a variety of scenarios [**2**]. For example, two coalescing resonance states typically results in power law-exponential decay of the form $P(t) \simeq (1 + C_1 t + C_2 t^2) e^{-\Gamma t}$ [**2, 3**]. However, in the case that the exceptional point occurs near the continuum threshold, this picture can be complexified in a number of scenarios, including pure power law decay with fractional exponents [**2, 4**] and anomalous-order EPs [**4**].

[1] S. Garmon, T. Petrosky, L. Simine and D. Segal, Fortschr. Phys. **61**, 261 (2013).
[2] S. Garmon, T. Sawada, K. Noba, and G. Ordonez, J. Phys.: Conf. Ser. **2038**, 012011 (2021).
[3] B. Dietz, et al, Phys. Rev. E **75**, 027201 (2007).
[4] S. Garmon, G. Ordonez, and N. Hatano, Phys. Rev. Res. **3**, 033029 (2021).