Introduction to Lieb-Schultz-Mattis theorem and Symmetry-Indicator Method

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Quantum many-body systems can realize a wide variety of phases, including symmetry-broken phases, symmetry-protected topological phases, and topologically ordered phases. The classification of possible phases of matter has advanced dramatically in recent decades. Historically, the different phases have been classified based on symmetry breaking, but in recent years, the importance of the topological perspective has become better understood.

In order to understand such complex systems in a unified manner, powerful insights can be gained by focusing on the symmetry of the system. The Lieb-Schultz-Mattis theorem and its generalizations are general criteria for available phases, telling us when a ground state can be unique and excitations are gapped at the same time. If this condition is not satisfied, the system must exhibit spontaneous symmetry breaking, topological order, or gapless excitations. The symmetry indicator method is another general approach that helps us determine if a given band structure is topologically nontrivial. This method can be applied to topological insulators, topological superconductors, and topological semimetals. I will spend most of the lecture time reviewing these ideas based on my book [1].

If time permits, I will also discuss our latest results towards a general understanding of possible phases of matter. The first story will be about topologically ordered phases without topological degeneracy [2]. The second is the possibility of spontaneous breaking of continuous symmetry in one dimension, which people may consider impossible based on the Hohenberg-Mermin-Wagner theorem [3].

References

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