Lieb-Robinson bound and its applications

One of the most profound questions in physics is to understand the universal structure of an assembly of many particles (i.e., quantum many-body system). Many of the unsolved problems in physics are related to quantum many-body physics: for example, high-temperature superconductivity, the origin of ferromagnetism, spin liquids, and topological order. Then, how should we describe the universal laws in quantum many-body physics? One answer is given in terms of information theory. The new field called Hamiltonian complexity aims to identify fundamental information-theoretic constraints imposed on quantum many-body systems. It significantly proceeds the systematic construction and classification of the quantum many-body systems (e.g., tensor network formalism).

A fundamental principle of many-body physics is causality: a strict prohibition of information propagation outside the light cone. However, in non-relativistic systems, it is often unclear whether such a light cone can be well defined. In the famous work by Lieb and Robinson, the amount of information is proved to be restricted in the effective light cone, which is characterized by the so-called "Lieb-Robinson bound." So far, the Lieb-Robinson bound is a crucial concept in accessing the precision error of various quantum simulation algorithms [1] and the mathematical structure of quantum entanglement at low (or zero) temperatures [2,3].

However, the original work by Lieb and Robinson and the followed generalizations are severely limited to systems with the two conditions [2], i.e., i) short-rangeness of the interactions and ii) finite bound of local energy. Although the breakdown of these conditions is ubiquitous in realistic experiments (e.g., cold atom setups), the speed limit on the information propagation has been unknown in such cases.

In the present talk, I would like to talk about the basics of the Lieb-Robinson bound, its applications, and recent progress on how to overcome the breakdown of the above two conditions [4,5,6,7,8].

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