

# Hyperlinked references for the lecture “Kardar-Parisi-Zhang universality class” in SQP2022

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- [1] Club De Montaña Calahorra, No title, Facebook.
  - [2] K. A. Takeuchi and M. Sano, Universal fluctuations of growing interfaces: Evidence in turbulent liquid crystals, *Phys. Rev. Lett.* **104**, 230601 (2010).
  - [3] K. A. Takeuchi, M. Sano, T. Sasamoto, and H. Spohn, Growing interfaces uncover universal fluctuations behind scale invariance, *Sci. Rep.* **1**, 34 (2011).
  - [4] K. A. Takeuchi and M. Sano, Evidence for geometry-dependent universal fluctuations of the Kardar-Parisi-Zhang interfaces in liquid-crystal turbulence, *J. Stat. Phys.* **147**, 853 (2012).
  - [5] P. J. Yunker, T. Still, M. A. Lohr, and A. G. Yodh, Suppression of the coffee-ring effect by shape-dependent capillary interactions, *Nature* **476**, 308 (2011).
  - [6] P. J. Yunker, M. A. Lohr, T. Still, A. Borodin, D. J. Durian, and A. G. Yodh, Effects of particle shape on growth dynamics at edges of evaporating drops of colloidal suspensions, *Phys. Rev. Lett.* **110**, 035501 (2013).
  - [7] M. A. C. Huergo, M. A. Pasquale, P. H. González, A. E. Bolzán, and A. J. Arvia, Growth dynamics of cancer cell colonies and their comparison with noncancerous cells, *Phys. Rev. E* **85**, 011918 (2012).
  - [8] A. Scheie, N. E. Sherman, M. Dupont, S. E. Nagler, M. B. Stone, G. E. Granroth, J. E. Moore, and D. A. Tennant, Detection of Kardar-Parisi-Zhang hydrodynamics in a quantum heisenberg spin-1/2 chain, *Nat. Phys.* **17**, 726 (2021).
  - [9] D. Wei, A. Rubio-Abadal, B. Ye, F. Machado, J. Kemp, K. Srakaew, S. Hollerith, J. Rui, S. Gopalakrishnan, N. Y. Yao, I. Bloch, and J. Zeiher, Quantum gas microscopy of Kardar-Parisi-Zhang superdiffusion, *Science* **376**, 716 (2022).
  - [10] K. A. Takeuchi, An appetizer to modern developments on the Kardar-Parisi-Zhang universality class, *Physica A* **504**, 77 (2018).
  - [11] 竹内一将, KPZ 普遍クラス – 厳密解と実験が奏でる非平衡のスケーリング則, *物性研究・電子版* **8**, 081205 (2020).

- [12] A.-L. Barabási and H. E. Stanley, *Fractal Concepts in Surface Growth* (Cambridge Univ. Press, Cambridge, 1995).
- [13] T. Halpin-Healy and Y.-C. Zhang, Kinetic roughening phenomena, stochastic growth, directed polymers and all that. aspects of multidisciplinary statistical mechanics, *Phys. Rep.* **254**, 215 (1995).
- [14] J. Krug, Origins of scale invariance in growth processes, *Adv. Phys.* **46**, 139 (1997).
- [15] I. Corwin, The Kardar-Parisi-Zhang equation and universality class, *Random Matrices Theory Appl.* **1**, 1130001 (2012).
- [16] J. Quastel and H. Spohn, The one-dimensional KPZ equation and its universality class, *J. Stat. Phys.* **160**, 965 (2015).
- [17] T. Kriecherbauer and J. Krug, A pedestrian's view on interacting particle systems, KPZ universality and random matrices, *J. Phys. A* **43**, 403001 (2010).
- [18] T. Sasamoto, The 1d Kardar-Parisi-Zhang equation: Height distribution and universality, *Prog. Theor. Exp. Phys.* **2016**, 022A01 (2016).
- [19] 笹本智弘, KPZ ユニバーサリテイクラス, *物性研究・電子版* **4**, 044209 (2015).
- [20] H. Spohn, Fluctuating hydrodynamics approach to equilibrium time correlations for anharmonic chains, in *Thermal Transport in Low Dimensions*, Lecture Notes in Physics, Vol. 921, edited by S. Lepri (Springer, Heidelberg, 2016) Chap. 3, pp. 107–158, arXiv:1505.05987.
- [21] M. Kardar, G. Parisi, and Y.-C. Zhang, Dynamic scaling of growing interfaces, *Phys. Rev. Lett.* **56**, 889 (1986).
- [22] W. A. Woyczyński, *Burgers-KPZ Turbulence*, Lecture Notes in Mathematics, Vol. 1700 (Springer, 1998).
- [23] C. W. Gardiner, *Handbook of Stochastic Methods for Physics, Chemistry and the Natural Sciences*, 3rd ed., Springer Series in Synergetics (Springer, Berlin, 2004).
- [24] L. Bertini and G. Giacomin, Stochastic Burgers and KPZ equations from particle systems, *Commun. Math. Phys.* **183**, 571 (1997).
- [25] T. Sasamoto and H. Spohn, One-dimensional Kardar-Parisi-Zhang equation: An exact solution and its universality, *Phys. Rev. Lett.* **104**, 230602 (2010).
- [26] C.-H. Lam and F. G. Shin, Improved discretization of the Kardar-Parisi-Zhang equation, *Phys. Rev. E* **58**, 5592 (1998).
- [27] M. Hairer, Solving the KPZ equation, *Ann. Math.* **178**, 559 (2013).
- [28] M. Hairer, A theory of regularity structures, *Invent. Math.* **198**, 269 (2014).
- [29] K. Johansson, Shape fluctuations and random matrices, *Commun. Math. Phys.* **209**, 437 (2000).
- [30] M. Prähofer and H. Spohn, Universal distributions for growth processes in 1 + 1 dimensions and random matrices, *Phys. Rev. Lett.* **84**, 4882 (2000).
- [31] C. A. Tracy and H. Widom, Asymptotics in asep with step initial condition, *Commun. Math. Phys.* **290**, 129 (2009).
- [32] T. Sasamoto and H. Spohn, Exact height distributions for the KPZ equation with narrow wedge initial condition, *Nucl. Phys. B* **834**, 523 (2010).
- [33] G. Amir, I. Corwin, and J. Quastel, Probability distribution of the free energy of the continuum directed random polymer in 1 + 1 dimensions, *Commun. Pure Appl. Math.* **64**, 466 (2011).
- [34] P. Calabrese, P. Le Doussal, and A. Rosso, Free-energy distribution of the directed polymer at high temperature, *Europhys. Lett.* **90**, 20002 (2010).
- [35] V. Dotsenko, Bethe ansatz derivation of the Tracy-Widom distribution for one-dimensional directed polymers, *Europhys. Lett.* **90**, 20003 (2010).
- [36] C. A. Tracy and H. Widom, Level-spacing distributions and the airy kernel, *Commun. Math. Phys.* **159**, 151 (1994).
- [37] C. A. Tracy and H. Widom, On orthogonal and symplectic matrix ensembles, *Commun. Math. Phys.* **177**, 727 (1996).
- [38] M. L. Mehta, *Random Matrices*, 3rd ed., Pure and Applied Mathematics, Vol. 142 (Elsevier, San Diego, 2004).
- [39] G. Akemann, J. Baik, and P. D. Francesco, eds., *The Oxford Handbook of Random Matrix Theory*, Oxford Handbooks (Oxford University Press, Oxford, 2015).
- [40] 永尾太郎, *ランダム行列の基礎* (東京大学出版会, 東京, 2005).
- [41] F. Bornemann, On the numerical evaluation of fredholm determinants, *Math. Comput.* **79**, 871 (2010).
- [42] F. Bornemann, On the numerical evaluation of distributions in random matrix theory: A review, *Markov Processes Relat. Fields* **16**, 803 (2010).
- [43] R. M. Conte and M. Musette, *The Painlevé Handbook* (Springer, Dordrecht, 2008).
- [44] M. Prähofer and H. Spohn, Exact scaling functions for one-dimensional stationary KPZ growth, webpage (2003).
- [45] J. Baik and E. M. Rains, Limiting distributions for a polynuclear growth model with external sources, *J. Stat. Phys.* **100**, 523 (2000).
- [46] J. Quastel and D. Remenik, Airy processes and variational problems, in *Topics in Percolative and Disordered Systems*, Springer Proceedings in Mathematics & Statistics, Vol. 69, edited by A. Ramírez, G. Ben Arous, P. Ferrari, C. Newman, V. Sidoravicius, and M. Vares (Springer, New York, 2014) pp. 121–171, arXiv:1301.0750.
- [47] M. Prähofer and H. Spohn, Scale invariance of the PNG droplet and the Airy process, *J. Stat. Phys.* **108**, 1071 (2002).
- [48] T. Sasamoto, Spatial correlations of the 1D KPZ surface on a flat substrate, *J. Phys. A* **38**, L549 (2005).
- [49] J. Baik, P. L. Ferrari, and S. Péché, Limit process of stationary TASEP near the characteristic line, *Commun. Pure Appl. Math.* **63**, 1017 (2010).
- [50] M. Prähofer and H. Spohn, Exact scaling functions for one-dimensional stationary KPZ growth, *J. Stat. Phys.* **115**, 255 (2004).
- [51] T. Iwatsuka, Y. T. Fukai, and K. A. Takeuchi, Direct evidence for universal statistics of stationary Kardar-Parisi-Zhang interfaces, *Phys. Rev. Lett.* **124**, 250602 (2020).
- [52] B. Derrida, An exactly soluble non-equilibrium system: The asymmetric simple exclusion process, *Phys. Rep.* **301**, 65 (1998).
- [53] O. Golinelli and K. Mallick, The asymmetric simple exclusion process: an integrable model for non-equilibrium statistical

- mechanics, *J. Phys. A* **39**, 12679 (2006).
- [54] P. Calabrese and P. Le Doussal, Exact solution for the Kardar-Parisi-Zhang equation with flat initial conditions, *Phys. Rev. Lett.* **106**, 250603 (2011).
  - [55] Y. T. Fukai and K. A. Takeuchi, Kardar-Parisi-Zhang interfaces with curved initial shapes and variational formula, *Phys. Rev. Lett.* **124**, 060601 (2020).
  - [56] O. A. Castro-Alvaredo, B. Doyon, and T. Yoshimura, Emergent hydrodynamics in integrable quantum systems out of equilibrium, *Phys. Rev. X* **6**, 041065 (2016).
  - [57] B. Doyon, Lecture notes on generalised hydrodynamics, *SciPost Phys. Lect. Notes* **18**, 1 (2020).
  - [58] M. Ljubotina, M. Žnidarič, and T. Prosen, Spin diffusion from an inhomogeneous quench in an integrable system, *Nat. Commun.* **8**, 16117 (2017).
  - [59] M. Ljubotina, M. Žnidarič, and T. Prosen, Kardar-Parisi-Zhang physics in the quantum Heisenberg magnet, *Phys. Rev. Lett.* **122**, 210602 (2019).
  - [60] I. Bloch, J. Dalibard, and S. Nascimbène, Quantum simulations with ultracold quantum gases, *Nat. Phys.* **8**, 267 (2012).
  - [61] Ž. Krajnik, E. Ilievski, and T. Prosen, Absence of normal fluctuations in an integrable magnet, *Phys. Rev. Lett.* **128**, 090604 (2022).
  - [62] V. B. Bulchandani, Kardar-Parisi-Zhang universality from soft gauge modes, *Phys. Rev. B* **101**, 041411 (2020).
  - [63] V. B. Bulchandani, S. Gopalakrishnan, and E. Ilievski, Superdiffusion in spin chains, *J. Stat. Mech.* **2021**, 084001 (2021).
  - [64] S. Gopalakrishnan and R. Vasseur, Kinetic theory of spin diffusion and superdiffusion in  $XXZ$  spin chains, *Phys. Rev. Lett.* **122**, 127202 (2019).