Tensor Network Formulation
- Basic Concepts and Applications -

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Tensor network (TN) formulation has been applied to a variety of problems in statistical and quantum physics. In this lecture we start from the basic concepts and elementary structure of TN. In particular, we focus on the matrix product (MP), which satisfies important aspects of TN, which are the representability of a wide class of states and the computability in the applications to practical problems. It should be noted that a large portion of states we consider in physics, such as low-energy states and thermal equilibrium states, are less entangled. This physical property enables us to precisely approximate, or sometimes exactly represent, those states under consideration. The TN study starts from believing in such a representability, although it should be verified afterward; the state you'd like to analyze can be expressed by TN.

The TN is constructed by means of contracting tensors. Thus there are so many adjustable parameters, and this is the reason why TN has a high potential of approximating a wide class of states. On the other hand, there are two many parameters to be optimized, and therefore computational treatment is avoidable in the TN formulation. There are several numerical algorithms for this purpose. The density matrix renormalization group (DMRG) is a well known example, which optimizes the MP part by part successively. The numerical processes in DMRG can be naturally understood from the variational principle with respect to the MP state (MPS). Within a limited time, let us glance at other representative algorithms, such as corner-transfer matrix (CTM) formalism, time evolving block decimation (TEBD) scheme, multiscale entanglement renormalization Ansatz (MERA), etc.

Probably we have some time to see applications of TN formalism to correlated systems. A big progress has been achieved by the TN renormalization (TNR), where the flow toward the fixed point in renormalization group (RG) is correctly captured. I conjecture that, some time in far (?) future, a sort of layered TN enables us to express a macroscopic system that contains classical information, from the view point of quantum mechanics in microscopic world.

References

[1] Please access the following web page to capture the daily developments in this field: http://quattro.phys.sci.kobe-u.ac.jp/dmrg/condmat.html