Two-body wave functions and compositeness from scattering amplitudes

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For a general two-body bound state in quantum mechanics, both in the stable and decaying cases, we establish a way to extract its two-body wave function in momentum space from the scattering amplitude of the constituent two particles.

For this purpose, we first show that the two-body wave function of the bound state corresponds to the residue of the off-shell scattering amplitude at the bound state pole, by considering solutions of both the Schrödinger and Lippmann–Schwinger equations. This means that solving the Lippmann–Schwinger equation at the bound state pole is equivalent to evaluating the two-body wave function of the bound state.

Then, we examine our scheme in several schematic models and extract the two-body wave function from the scattering amplitudes. We consider both stable and unstable bound states, and in single-channel and coupled-channels problems. With the schematic models we discuss general properties of the two-body wave function and compositeness for bound states. Of special interest is that we obtain the normalized two-body wave function of the bound state from the scattering amplitude, regardless of whether the bound state is stable or not; the norm of the two-body wave function, to which we refer as the compositeness, is unity for an energy independent interaction, while the compositeness deviates from unity for an energy dependent interaction, which can be interpreted to implement missing channel contributions.

As applications, we investigate the compositeness of several hadronic resonances. With our scheme, we may able to distinguish hadronic molecules, which are composed of two (or more) hadrons themselves, like deuteron as a proton–neutron bound state, rather than compact qqq states for baryons and $q\bar{q}$ for mesons. We evaluate the compositeness for candidates of hadronic molecules such as $\Lambda(1405)$ from hadron–hadron scattering amplitudes in hadron effective models.