Random Green Matrices: from Proximity Resonances to Anderson Localization

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Universal properties of the spectra of certain non-Hermitian random matrices with strongly correlated elements are discovered. Such matrices can describe multiple elastic scattering of waves from a collection of point-like objects. In this case the elements of these matrices are proportional to the free-space Green's function calculated for the differences between positions of any pair of scatterers. Striking physical interpretation of the obtained results within Breit-Wigner's model of the single scatterer is elaborated. In this case the eigenvalues of the random Green matrix can be considered as a first approximation to the resonance poles of an open system. Proximity resonances and Anderson localization are considered as two illustrative examples.

The spectrum of the Green matrix with uncorrelated phases resembles the results from the theory of non-Hermitian random matrices — it forms a disk on a complex plane (with added tails corresponding to the proximity resonances). The disk is centered at zero and its size grows with the growing size of the matrix. Thus in the limit of an infinite system the eigenvalues are expected to fill the whole complex plane. Addition of correlations changes the spectrum in a dramatic way — it is no longer symmetric with respect to the point. Instead all eigenvalues are shifted to a half-plane. Another phase transition in the spectrum appears for the increasing size of the system. It can be understood as Anderson localization of classical waves in a system of randomly distributed scatterers.

Systems of 10^4 randomly distributed scalar scatterers are studied numerically. It is shown that in the localization regieme resonace widths obey a power law distribution $P(\Gamma) \propto \Gamma^{-1}$. The same result has been obtained diagonalization of the hamiltonian from the three-dimensional Anderson model. Our calculations of disordered three-dimensional media yield a Poisson distribution of level spacings $P(\Delta E) \propto \exp(-\Delta E)$. Nonlinear sigma model gives a similar result for an one-dimensional disordered medium in the localization regieme. For increasing size of the system the second distribution decreases slower than the first. Thus more and more resonance poles fullfill the Thouless criterion of localization $\Delta E > \Gamma$.

In our studies prelocalized modes are identified as resonance poles satisfying the Thouless criterion of localization. First prelocalized modes appear for scatterer densities lower than the critical density predicted by the Ioffe-Regel criterion $k\,l < 1$. It is shown that the band of localized waves, emerging in the limit of an infinite medium is a little bit wider than the theoretical width predicted by the Ioffe-Regel criterion. Another surprising result of our calculations is that the band of localized waves does not contain the resonance energy of a single scatterer! Experiments on scattering of sound waves on bubbles in water also seem to reveal some localization effects at frequencies above the resonant frequency of a single bubble.