Pseudomode quantum jump and memory effect of non-Markovian dynamics

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We study non-Markovian dynamics of a two level atom, using pseudomode method [1]. Pseudomode method is one of the methods treating non-Markovian dynamics. With this method, non-Markovian dynamics of the system of interest can be mapped to Markovian dynamics of a combined system of the system and pseudomodes. So we can apply methods for Markovian dynamics to the combined system. The purpose of this work is that we get a physically intuitive insight into the memory effect of non-Markovian dynamics.

The Hamiltonian for our system is

$$H = \frac{\hbar\omega_0}{2}\sigma_z + \sum_k \hbar\omega_k b_k^\dagger b_k + \sum_k \hbar g_k \left(\sigma_+ b_k + \sigma_- b_k^\dagger\right), \tag{1}$$

and the initial state of the reservoir is vacuum state. For simplicity, we consider that the density of states is described by a frequency dependent function $D(\omega)$ which is a sum of Lorentzians as follows

$$D(\omega) = \sum_{l=1}^{L} \frac{W_l \Gamma_l}{(\omega - \omega_l)^2 + (\Gamma_l/2)^2}, \tag{2}$$

and $\omega_l, W_l, \Gamma_l > 0$. With quantum jump approach [2], the probability density of a photon emission from the combined system is $p(t) = -\frac{d}{dt}\langle\tilde{\Psi}(t)|\tilde{\Psi}(t)\rangle$. Here $|\tilde{\Psi}(t)\rangle$ is a single quantum trajectory governed by a Schrödinger equation with a non-Hermitian Hamiltonian,

$$H_{\text{eff}} = \frac{\hbar\omega_0}{2}\sigma_z + \sum_{l=1}^{L}\left(\hbar\omega_l - i\frac{\hbar\Gamma_l}{2}\right) c_l^\dagger c_l + \sum_{l=1}^{L} \hbar\sqrt{W_l}\,\Omega\left(\sigma_+ c_l + \sigma_- c_l^\dagger\right), \tag{3}$$

where $c_l^\dagger$ and c_l are the bosonic creation and annihilation operators for the pseudomode labeled by l and Ω is a strength of the total coupling $\Omega^2 = \sum_k g_k^2$.

We discuss about the dynamics of $p(t)$ and the expectation value of jump time $\langle t\rangle$. The expectation value $\langle t\rangle$ can be divided into $\langle t\rangle_S$ and a sum of $\langle t\rangle_l$, which are the expected time length that states of the system and pseudomode l are in their excited state, respectively. In Markovian limit, $\langle t\rangle_l$ and $\langle t\rangle_S$ converge to 0 and the same value of Markovian dynamics of a two level atom, respectively. The Markovian approximation means that the reservoir has no memory so that we can infer that $\langle t\rangle_l$ reflects the memory effect of non-Markovian dynamics. In particular, we consider the damped Jaynes Cummings model, which is a model of a two level atom in a lossy cavity.

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