Distribution of the S-matrix poles in the radial Scarf II and the generalized Woods–Saxon potentials

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The generalized Woods–Saxon (GWS) potential has been used extensively in nuclear reaction and structure calculations. For $l = 0$ the S-matrix of this potential can be determined analytically [1], while computer codes are available for numerical calculations for higher partial waves. In numerical studies a finite cut-off distance has to be applied, and this unphysical parameter has strong infulence on the poles of this CGWS potential. The analytical results for $l = 0$ help to reveal these effects.

In the case of the GWS potential the wave functions are reflected at the nuclear radius, therefore the distance of the resonant poles depends on the radius parameter. In the CGWS potentials the wave function can also be reflected at larger distance, where the potential is cut to zero. The position of the resonant poles do depend strongly on the cut-off distance, except for a few narrow resonances in potentials with barrier. When a strong barrier is applied in the CGWS potential, then three groups of resonances appear. The first group is produced by the reflection on the potential barrier, while the second one is due to the reflection at the cut-off distance. The few resonances in the third group are due to the double reflection at the barrier and at the cut-off distance. In the case without cut-off (i.e. the GWS), the poles are due to the reflections at the nuclear radius.

This procedure can also be applied to the radial Scarf II potential. This potential has been studied extensively as a one-dimensional problem, and its transmission and reflection amplitudes are known in an analytical form [1]. It can be converted into a radial problem with $l = 0$ if it is cut at a position that is defined to be the origin. In this case the physical solutions are constructed from the linear combination of the two independent solutions of the Schrödinger equation, requiring that it vanishes at the origin. Then the S-matrix is determined from the asymptotic behavior of the solution. This is exactly the same procedure that has been applied to obtain the S-matrix of the GWS potential for $l = 0$. Furthermore, this latter potential can be recognized as the shifted and cut version of the one-dimensional Rosen–Morse II potential. Additionally, the Scarf II and Rosen–Morse II potentials have a common term ($\sim [\cosh(ax)]^{-2}$) and their difference shows up in their second term ($\sim \sinh(ax)[\cosh(ax)]^{-2}$, and $\sim \tanh(ax)$, respectively).

Numerical and analytical calculations similar to those for the GWS and CGWS potentials have been carried out for the radial Scarf II potential and its special version that has been cut at a finite distance. The results are qualitatively similar to those outlined above for the GWS and CGWS potentials.

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