# Degenerate two-body and three-body coupled-channels system and the $S$-matrix pole behavior near the thresholds 

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Motivated by the existence of candidates for the exotic hadron whose masses are close to both of two-body and three-body hadronic thresholds, we investigated the $S$-matrix pole behavior near the thresholds in a degenerate two-body and three-body coupledchannels system.

Two-body and three-body coupled-channels scattering equations are formulated as effective three-body scattering equations in which effects induced by a coupling to the twobody channel are embedded as effective interactions in the three-body channel constructed by the Feshbach projection method. We solve the eigenvalue equations of the kernel of the scattering equations instead of the scattering equations themselves to obtain the $S$ matrix pole. Although the transition amplitudes have physical singularities, the kernel of the scattering equations does not, i.e. it has unphysical singularities. However, we show that this unphysical singularity problem can be resolved by an appropriate reorganization of the scattering equations and the mass renormalization.

The $S$-matrix pole behavior near the thresholds in the degenerate two-body and threebody coupled-channels system has universal property whose pole energy $E$ is determined by an equation $g-E \log (-E)=0$, that is, it is determined by one parameter $g$. The behavior is neither those of a single-channel two-body nor a three-body system. We expect the behavior is realized also for those candidates for the exotic hadrons that lie around the energy regions where two-body as well as three-body thresholds rest close to each other and play a key role in understanding those resonances.

