SEIKEN SYMPOSIUM No. 88

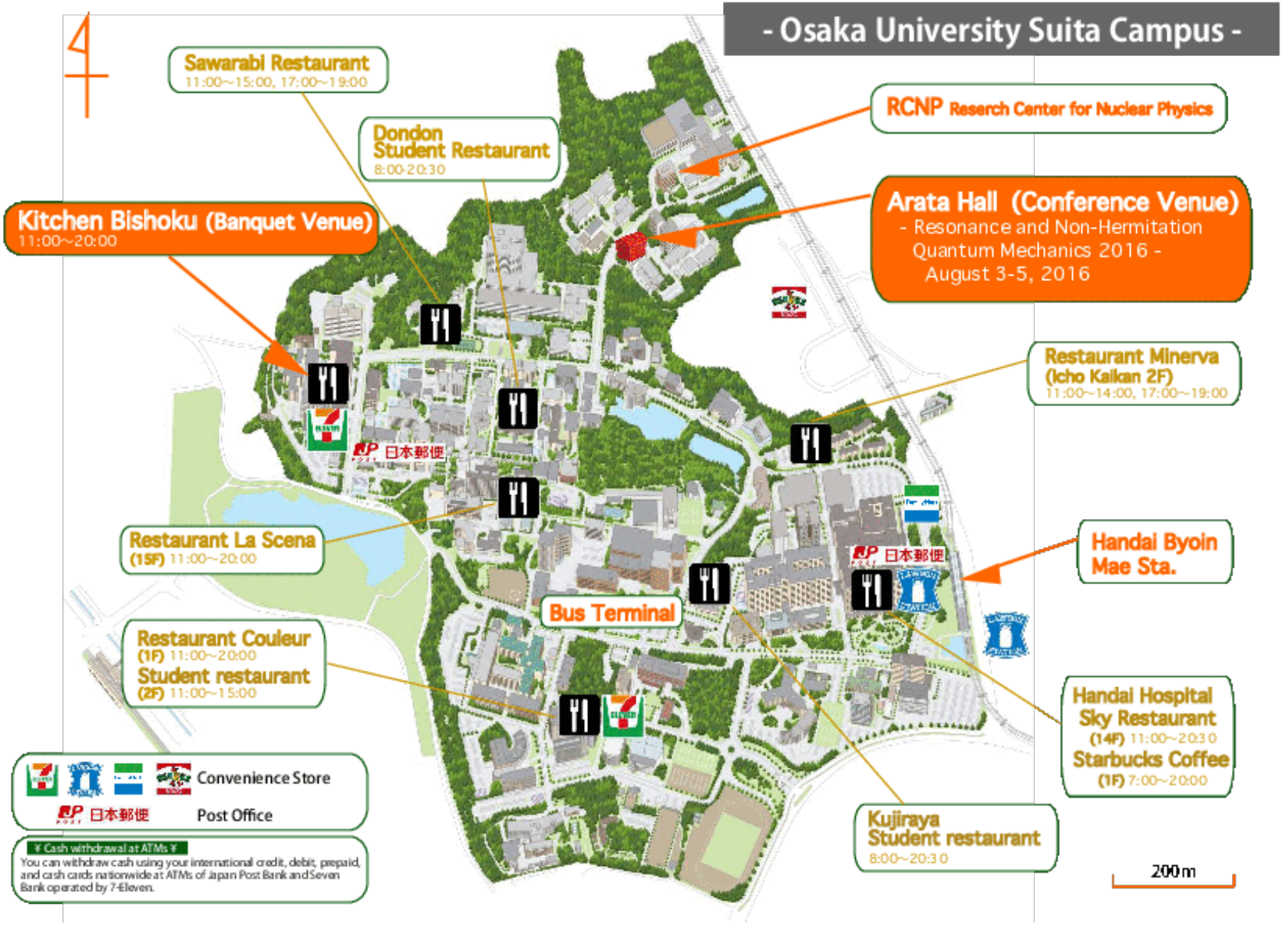
Resonance
and
Non-Hermitian Quantum Mechanics 2016

August 3–5, 2016

Research Center for Nuclear Physics (RCNP), Osaka University

Abstract Book

- Osaka University Suita Campus -



August 3		August 4		August 5	
		Session QM4			
9:00	registration	9:00	Ordonez		
9:20	opening				
Session QM1				Session QM7	
9:30	Feinberg			9:30	Petrosky
		9:45	Shimasaki		
10:15	Rusek			10:15	Hashimoto
10:35	Levai	10:30	lab tour	10:35	Garmon
10:55	break			10:55	break
11:05	Takayanagi			11:10	Tanaka
				11:30	Noba
11:50	Heiss			11:50	Kim
12:10	Siegl			12:10	lunch
12:30	lunch	12:30	lunch		
Session NP2		Session NP5		Session NP8	
13:50	Michel	13:50	Lazauskas	13:50	Dote
				14:10	Sekihara
14:35	Masui	14:35	Ito	14:30	Konishi
14:55	break	14:55	break	14:50	Miyahara
15:10	Myo	15:10	Mizutori	15:10	break
15:30	Kikuchi	15:30	Kobayashi	Session QM9	
15:50	Kato	15:50	break	15:25	Znojil
16:10	break	Session QM6		15:45	Fukuta
Session QM3		16:05	Morishita	16:05	Yamane
16:25	Yuasa			16:25	Dunham
		16:50	Luna-Acosta	16:45	Ganguly
17:10	Ohyama	17:10	Melgaard	17:05	closing
17:30	Ishkhanyan	17:30	Kumar		
17:50	Yadav	17:50			
18:10					
18:30	banquet				

Statistics of Resonances in One-Dimensional Continuous Systems - Explicit Results

Joshua Feinberg
Physics Department, University of Haifa at Oranim, and Technion
joshua@physics.technion.ac.il

We study the average density of resonances (DOR) of a disordered one-dimensional continuous open system. The disordered system is semi-infinite, with white-noise random potential, and it is coupled to the external world by a semi-infinite continuous perfect lead. We use a previously obtained integral representation for the DOR, which involves the probability density function of the logarithmic derivative of the wave function at the contact point, to obtain the DOR explicitly.

Random Green Matrices: from Proximity Resonances to Anderson Localization

Marian Rusek
Warsaw University of Life Sciences, Poland
marian_rusek@sggw.pl

Universal properties of the spectra of certain non-Hermitian random matrices with strongly correlated elements are discovered. Such matrices can describe multiple elastic scattering of waves from a collection of point-like objects. In this case the elements of these matrices are proportional to the free-space Green's function calculated for the differences between positions of any pair of scatterers. Striking physical interpretation of the obtained results within Breit-Wigner's model of the single scatterer is elaborated. In this case the eigenvalues of the random Green matrix can be considered as a first approximation to the resonance poles of an open system. Proximity resonances and Anderson localization are considered as two illustrative examples.

The spectrum of the Green matrix with uncorrelated phases resembles the results from the theory of non-Hermitian random matrices — it forms a disk on a complex plane (with added tails corresponding to the proximity resonances). The disk is centered at zero and its size grows with the growing size of the matrix. Thus in the limit of an infinite system the eigenvalues are expected to fill the whole complex plane. Addition of correlations changes the spectrum in a dramatic way — it is no longer symmetric with respect to the point. Instead all eigenvalues are shifted to a half-plane. Another phase transition in the spectrum appears for the increasing size of the system. It can be understood as Anderson localization of classical waves in a system of randomly distributed scatterers.

Systems of 10^4 randomly distributed scalar scatterers are studied numerically. It is shown that in the localization regieme resonace widths obey a power law distribution $P(\Gamma) \propto \Gamma^{-1}$. The same result has been obtained diagonalization of the hamiltonian from the three-dimensional Anderson model. Our calculations of disordered three-dimensional media yield a Poisson distribution of level spacings $P(\Delta E) \propto \exp(-\Delta E)$. Nonlinear sigma model gives a similar result for an one-dimensional disordered medium in the localization regieme. For increasing size of the system the second distribution decreases slower than the first. Thus more and more resonance poles fullfill the Thouless criterion of localization $\Delta E > \Gamma$.

In our studies prelocalized modes are identified as resonance poles satisfying the Thouless criterion of localization. First prelocalized modes appear for scatterer densities lower than the critical density predicted by the Ioffe-Regel criterion $k\,l < 1$. It is shown that the band of localized waves, emerging in the limit of an infinite medium is a little bit wider than the theoretical width predicted by the Ioffe-Regel criterion. Another surprising result of our calculations is that the band of localized waves does not contain the resonance energy of a single scatterer! Experiments on scattering of sound waves on bubbles in water also seem to reveal some localization effects at frequencies above the resonant frequency of a single bubble.

Distribution of the S-matrix poles in the radial Scarf II and the generalized Woods–Saxon potentials

G. Lévai[1], Á. Baran[2], P. Salamon[1], T. Vertse[1,2]
Institute for Nuclear Research, Hungarian Academy of Sciences (MTA Atomki), Debrecen, Hungary[1], Faculty of Informatics, University of Debrecen, Hungary[2]
levai@atomki.mta.hu[1]

The generalized Woods–Saxon (GWS) potential has been used extensively in nuclear reaction and structure calculations. For $l = 0$ the S-matrix of this potential can be determined analytically [1], while computer codes are available for numerical calculations for higher partial waves. In numerical studies a finite cut-off distance has to be applied, and this unphysical parameter has strong infulence on the poles of this CGWS potential. The analytical results for $l = 0$ help to reveal these effects.

In the case of the GWS potential the wave functions are reflected at the nuclear radius, therefore the distance of the resonant poles depends on the radius parameter. In the CGWS potentials the wave function can also be reflected at larger distance, where the potential is cut to zero. The position of the resonant poles do depend strongly on the cut-off distance, except for a few narrow resonances in potentials with barrier. When a strong barrier is applied in the CGWS potential, then three groups of resonances appear. The first group is produced by the reflection on the potential barrier, while the second one is due to the reflection at the cut-off distance. The few resonances in the third group are due to the double reflection at the barrier and at the cut-off distance. In the case without cut-off (i.e. the GWS), the poles are due to the reflections at the nuclear radius.

This procedure can also be applied to the radial Scarf II potential. This potential has been studied extensively as a one-dimensional problem, and its transmission and reflection amplitudes are known in an analytical form [1]. It can be converted into a radial problem with $l = 0$ if it is cut at a position that is defined to be the origin. In this case the physical solutions are constructed from the linear combination of the two independent solutions of the Schrödinger equation, requiring that it vanishes at the origin. Then the S-matrix is determined from the asymptotic behavior of the solution. This is exactly the same procedure that has been applied to obtain the S-matrix of the GWS potential for $l = 0$. Furthermore, this latter potential can be recognized as the shifted and cut version of the one-dimensional Rosen–Morse II potential. Additionally, the Scarf II and Rosen–Morse II potentials have a common term ($\sim [\cosh(ax)]^{-2}$) and their difference shows up in their second term ($\sim \sinh(ax)[\cosh(ax)]^{-2}$, and $\sim \tanh(ax)$, respectively).

Numerical and analytical calculations similar to those for the GWS and CGWS potentials have been carried out for the radial Scarf II potential and its special version that has been cut at a finite distance. The results are qualitatively similar to those outlined above for the GWS and CGWS potentials.

[1] Gy. Bencze, *Commentationes Physico-Mathematicae* **31** (1966) 1.
[2] P. Salamon, Á. Baran, T. Vertse, *Nucl. Phys.* A **952** (2016) 1-17.
[3] G. Lévai, F. Cannata and A. Ventura, *J. Phys. A: Math. Gen.* **34** (2001) 839.

Theoretical analyses of transition state spectroscopy of chemical reactions: on the relation between transition state structure and quantum resonance

Toshiyuki Takayanagi
Department of Chemistry, Saitama University
tako@mail.saitama-u.ac.jp

The concept of "transition state" has been playing an essentially important role in understanding detailed mechanisms, dynamics and rates of chemical reactions. The transition state structure is frequently defined as a saddle point or a specific region around the saddle point on the potential energy surface. Direct spectroscopic detection of the transition-state is called "Transition-State Spectroscopy (TSS)". The measurement of the photodetachment spectrum of the molecular anion may be one of the most successful TSS methods. It has been shown that the measured photodetachment spectra can provide important information on the transition-state of the neutral system if the stable region of the anionic potential energy surface has good Franck-Condon overlap with the transition-state region of the neutral potential energy surface. However, the analysis of the measured spectra is not straightforward. This is simply because the observed spectral peaks are corresponding to quantum resonance states of chemical reactions. Thus, in order to extract detailed information on transition-state properties, one has to employ reliable theoretical methods that can describe the reaction dynamics at a quantum mechanical level. In this talk I would like to address the transition-state spectra of the Br + HBr and I + HI reactions. We have developed the potential energy surfaces using ab initio electronic structure calculations to study the transition-state spectra of these reactions. It was found that the simulated high-resolution photodetachment spectra show broad anti-symmetric progressions with many sharp resonances associated with van der Waals complex localized around potential energy wells. In addition, it was also found that there is the possibility that the BrMuBr and IMuI systems have vibrational bonding states, corresponding to true "bound" states whose wavefunctions are localized around the transition state region.

Three coupled wave guides and third order exceptional points

W Dieter Heiss[1], Guenter Wunner[2]
Department of Physics, University Stellenbosch, South Africa[1],
1st Institute of theoretical physics, University Stuttgart, Germany[2]
dieter@physics.sun.ac.za

A PT-symmetric model for three interacting wave guides is investigated. Each wave guide is represented by an attractive delta-function potential being in equi-distant positions. The two outer potentials are complex describing loss and gain, respectively. The real parts of the outer potentials are assumed to be equal. The major focus of the study lies on the occurrence of an exceptional point of third order and the physical effects of such singularity. While some results resemble those from similar studies with two wave guides, the three wave guides appear to have a richer structure. Emphasis is placed on the fine tuning in the approach of the EP3 as this appears to be a particular challenge for an experimental realization

Pseudospectra and Schrödinger operators with complex potentials

Petr Siegl
Mathematical Institute, University of Bern, Switzerland
petr.siegl@math.unibe.ch

The departure from the behavior of normal operators can be described by the notion of pseudospectra. In particular, the pseudospectral analysis reveals spectral (in)-stabilities or yields decay rates of the associated semigroup (time evolution). We summarize main facts on pseudospectra and explain how recent tools can be used for the analysis of Schrödinger operators with complex potentials.

The talk is partly based on

[1] P. Siegl and D. Krejčiřík, *On the metric operator for the imaginary cubic oscillator*, Physical Review D, 86, (2012) 121702(R).
[2] D. Krejčiřík, P. Siegl, M. Tater, and J. Viola, *Pseudospectra in non-Hermitian quantum mechanics*, Journal of Mathematical Physics, 56, (2015) 103513.

Gamow Shell Model: nuclear structure

Nicolas Michel
National Superconducting Cyclotron Laboratory, Michigan State University
michel@nscl.msu.edu

Nuclei situated at drip-lines exhibit different properties as those lying close the valley of stability. The ground states of drip-line nuclei can be very extended in space, so that they form a halo of nucleons in the classically forbidden zone, or can even be unbound by particle emission. As a fact, drip-line nuclei are open quantum systems, for which the close vicinity of the continuum of unbound scattering states must be taken into account theoretically.

For this, the Gamow Shell Model (GSM) has been introduced several years ago. Its fundamental idea is to generate a basis of many-body states from the one-body Berggren basis, comprising bound, resonant and scattering states. The continuum degrees of freedom are thus included at basis level, and the configuration mixing between many-body basis states takes care of inter-nucleon correlations.

Consequently, loosely bound and resonant nuclear many-body states can be calculated with GSM. In order to avoid center of mass excitations, GSM is defined in the Cluster Orbital Shell Model (COSM) frame, using a core and valence nucleons. The lack of exact antisymmetry in laboratory frame of COSM wave functions will be discussed and shown to be very small. Another issue is the identification of the many-body GSM resonant states from the diagonalization of the GSM matrix, situated in the middle of many-body scattering states. For this, the overlap method is used to identify GSM resonant many-body states from their pole approximation, along with the Davidson method, targeting interior GSM eigenvalues. However, for very large matrices, the Davidson method can no longer be used, so that the Density Matrix Renormalization Group (DMRG) has been introduced. Its fundamental idea is to build a basis of increasingly correlated many-body states, so that matrices bear tractable dimensions.

Applications will consist of the study of ^{6}He and ^{8}He charge radii, of experimental interest. GSM-COSM has also been tested against the Gaussian Expansion method for the case of ^{6}He and ^{6}Be. GSM-DMRG has been successfully applied to realistic interactions in the context of no core GSM, with the examples of ^{4}H, ^{4}Li, ^{5}He and ^{6}Li. Center of mass issues in this case will be discussed. The introduction in GSM and DMRG of natural orbitals, a basis tailored to the many-body state calculated, hence beyond the Hartree-Fock approximation, will also be presented.

Analysis of unbound-state contributions in the Borromean systems

Hiroshi Masui
Information Processing Center, Kitami Institute of Technology, Kitami, Japan
hgmasui@mail.kitami-it.ac.jp

The treatment of the unbound states has been discussed in the many different fields of quantum physics, for example, nuclear physics, atomic and molecular physics, and quantum optics. The essential difference between the bound and unbound states is the boundary condition of the wave function. Among the unbound states, resonant states have divergent behavior in the asymptotic region due to the complex momentum or energy. In order to treat the resonant states on the same footing to the bound states, we employ the complex scaling method (CSM) to transform the matrix element with the resonant wave function as the L^2-integrable. We apply CSM to the Gaussian expansion method [1] (GEM-CS), which is an efficient approach to describe the many-body wave function using the superposition of the Gaussian [2, 3].

There are alternative approaches for the treatment of the unbound states in many-body system. One of the successful approach for including the unbound states is the Gamow shell model (GSM)[4, 5]. In the approach, the basis sets are prepared with eigenstates of the single-particle states including the unbound states by extending the completeness relation. The guiding principle of the extension is to include the resonant poles by changing the contour path on the complex momentum plane.

Two approaches, GEM-CM and GSM are different techniques for describing the many-body wave function in the system, though the goal of both methods might be the same. Therefore, we perform the precise comparison between GEM-CS and GSM [6]. We discuss the results of the comparison and show the other related works [7].

References

[1] E. Hiyama, Y. Kino and M. Kamimura, Prog. Part. Nucl. Phys. **51**, 223 (2003).

[2] S. Aoyama, T. Myo, K. Katō, and K. Ikeda, Prog. Theor. Phys. **116**, 1 (2006).

[3] T. Myo, Y. Kikuchi, H. Masui, and K. Katō, Prog. Part. Nucl. Phys. **79**, 1 (2014)

[4] N. Michel, W. Nazarewicz, M. Płoszajczak and K. Bennaceur, Phys. Rev. Lett. **89**, 042502 (2002).

[5] N. Michel, W. Nazarewicz, M. Płoszajczak and J. Okołowicz, Phys. Rev. C **67**, 054311 (2003).

[6] H. Masui, K. Katō, N. Michel, M. Płoszajczak Phys. Rev. C **89**, 044317 (2014).

[7] A.T. Kruppa, G. Papadimitriou, W. Nazarewicz, N. Michel, Phys. Rev. C **89**, 014330 (2014).

Many-body resonances in light unstable nuclei using the complex scaling method

Takayuki Myo
General Education, Faculty of Engineering, Osaka Institute of Technology, Osaka, Osaka 535-8585, Japan
myo@ge.oit.ac.jp

Complex scaling method (CSM)[1, 2] is one of the approaches to study the resonances in many-body quantum system and often used in nuclear physics and hadron physics. For nuclear physics, in proton-rich and neutron-rich nuclei, most of the states are observed as unbound states owing to the weak binding nature of valence protons/neutrons surrounding the stable core nucleus. CSM becomes a powerful tool to investigate the properties of unbound states in those nuclei.

The resonance spectroscopy in unstable nuclei has been developed using radioactive-beam experiments [3]. In addition to the energies and decay widths of resonances, information on their configurations and spatial properties is important to understand the structures of unstable nuclei. We employ CSM for the description of the many-body resonances decaying into the system of three-body and more, which are often seen in light unstable nuclei [4]. In this contribution, we present our recent results with CSM.

1. Many-body resonances in neutron-rich He isotopes and their mirror proton-rich nuclei. We treat up to five-body resonances in those nuclei. We discuss the mirror symmetry breaking in the resonances between neutron-rich and proton-rich nuclei [5].

2. We present the complex-scaled Green's function to obtain the cross sections in the reaction using unstable nuclei. Green's function is essential to evaluate the cross sections not only of resonances, but also of non-resonant continuum part individually. We show the applications to the three-body breakup reactions of neutron-rich nuclei.

References

[1] J. Aguilar, J. M. Combes, Commun. Math. Phys. **22** (1971) 269.

[2] E. Balslev, J. M. Combes, Commun. Math. Phys. **22** (1971) 280.

[3] I. Tanihata, H. Savajols, R. Kanungo, Prog. Part. Nucl. Phys. **68** (2013) 215.

[4] T. Myo, Y. Kikuchi, H.Masui, K. Katō, Prog. Part. Nucl. Phys. **79** (2014) 1.

[5] T. Myo, K. Katō, Prog. Theor. Exp. Phys. (2014) 083D01.

Photodisintegration cross section of ^{9}Be in the complex-scaled $\alpha + \alpha + n$ three-body model

Y. Kikuchi[1,2], M. Odsuren[3], T. Myo[4,5], and K. Katō[6]
Department of Physics, Osaka City University[1], RIKEN Nishina Center[2],
School of Engineering and Applied Sciences, National University of Mongolia[3],
General Education, Faculty of Engineering, Osaka Institute of Technology[4],
Research Center for Nuclear Physics (RCNP), Osaka University[5],
Nuclear Reaction Data Centre, Faculty of Science, Hokkaido University[6]
yuma@sci.osaka-cu.ac.jp[1]

The photodisintegration cross section of ^{9}Be shows different features in different energy regions. In low energy region up to $E_\gamma = 6$ MeV, the cross section is dominated by the electromagnetic transitions into the resonances of ^{9}Be, and has been studied within the $\alpha + \alpha + n$ three-body model. On the other hand, the photodisintegration cross section in higher energy region shows a significant electric dipole strength, considered to be the transition into continuum states of $\alpha + \alpha + n$. It is interesting to discuss the mechanism of the photodisintegration of ^{9}Be in low and higher energy regions on the same footing.

In the present work, we investigate the photodisintegration cross section of ^{9}Be in low and higher energy regions by using the $\alpha + \alpha + n$ three-body model and the complex scaling method (CSM). The purposes of this work are following two: One is to investigate the excited states of ^{9}Be in low energy region connected with the ground state through the electromagnetic transitions. The other is to investigate the mechanism of the dipole transition in ^{9}Be in higher energy region. In the preset work, we employ the $\alpha + \alpha + n$ three-body model and calculate the photodisintegration cross section by applying the CSM to the $\alpha + \alpha + n$ three-body model.

In this contribution, we present our calculated photodisinegration cross section of ^{9}Be. Also, we discuss the mechanism of the dipole transition. Using CSM, we decompose the dipole strength into the contributions of each decay channel, and show that the transitions into the ^{8}Be(2^+) + n continuum states dominate the dipole strength in higher energy region.

Reference

[1] YK, M. Odsuren, T. Myo, and K. Katō, Phys. Rev. C**93**, 054605 (2016).

Resonance or virtual state causing cross-section peaks just above thresholds

Kiyoshi Katō[1], Yuma Kikuchi[2,3], Myagmarjav Odsuren[4], Takayuki Myo[5,6], Victor S. Vasilevsky[7] and Nurgali Takibayev[8]
Nuclear Reaction Data Centre, Faculty of Science, Hokkaido University[1],
Department of Physics, Osaka City University[2], *RIKEN Nishina Center*[3],
School of Engineering and Applied Sciences, National University of Mongolia[4],
General Education, Faculty of Engineering, Osaka Institute of Technology[5], *Research Center for Nuclear Physics (RCNP), Osaka University*[6],
Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine[7]
IETP, Department of Physics and Technology, al-Farabi Kazakh National University[8]
kato@nucl.sci.hokudai.ac.jp[1]

It is a long standing problem to determine its resonance energy and width of the first excited $1/2^+$ state of ^{9}Be, which is closely connected with the problem to clarify whether it is a resonant state or not. From analyses of the scattering length for the ^{8}Be+n scattering and the R-matrix calculations, it has been shown that the first excited $1/2^+$ state is a virtual state. On the other hand, recently the first excited $1/2^+$ state has been discussed as a three-body resonance of $\alpha + \alpha + n$.

Experimentally, the photodisintegration cross section of $^9\mathrm{Be}+\gamma \to \alpha + \alpha + n$ in a low energy region has been measured to deduce a production rate of ^{9}Be from the astrophysical point of view. In the low energy region up to $E_\gamma = 6$ MeV, the enhancement of the cross section has been observed at several energy positions corresponding to excited states of ^{9}Be, which are understood to be due to the electro-magnetic dipole transitions. In particular, the first excited $1/2^+$ state is observed as a sharp peak just above the $^8\mathrm{Be}(0^+)+n$ threshold.

Recently, we studied the $1/2^+$ state of ^{9}Be and the photodisintegration cross section applying the complex scaling method to the $\alpha+\alpha+n$ three-cluster model [1]. The results indicate that there is no sharp resonant state corresponding to the distinct peak observed just above the $^8\mathrm{Be}+n$ threshold in the photodisintegration cross section of ^{9}Be. However, the recent experimental data of the $1/2^+$ cross section can be well reproduced by the $\alpha + \alpha + n$ three-cluster model calculation. From these results, we discuss that the first excited $1/2^+$ state in ^{9}Be is a $^8\mathrm{Be}+n$ virtual state but not resonant one.

Reference

[1] M. Odsuren, Y. Kikuchi, T. Myo, M. Aikawa, and K. Katō, Phys. Rev. C92, 014322 (2015).

Universal Control Induced by Noise

Kazuya Yuasa
Department of Physics, Waseda University, Tokyo 169-8555, Japan
yuasa@waseda.jp

Very frequent measurement on a quantum system projects the system onto the subspaces in the Hilbert space specified by the measurement, and the dynamics of the system is confined within the subspaces, evolving unitarily there. This is known as the quantum Zeno effect (QZE), and the unitary dynamics within the subspaces (Zeno subspaces) is called the quantum Zeno dynamics (QZD).

In this talk, we provide another interesting effect of the QZE: the QZE can *exponentially* enrich the dynamics of a quantum system, so as for us to be able to perform universal quantum computation. Suppose that we are given two Hamiltonians which are commutative with each other. Even if we switch on and off the two Hamiltonians, we can realize only trivial dynamics, due to the commutativity. We can change the situation by the QZE: if we frequently measure a small part of the system to induce the QZD, the two Hamiltonians are projected by the frequent measurement and become noncommutative with each other. Moreover, the two projected Hamiltonians are enough to realize any unitary evolution within the Zeno subspaces. In other words, we are able to perform universal quantum computation within the Zeno subspaces. We prove that this effect is generally to be expected: almost any quantum dynamics becomes universal if it is observed very frequently.

The same effect can be induced by strong dissipative and/or dephasing process. In particular, it can turn trivial control into universal one, at the same time realizing decoherence-free subspaces. We can also do the opposite: any complex quantum dynamics can be viewed as a projected dynamics of a trivial dynamics in an extended Hilbert space. The Hamiltonians can be "purified" to make them commutative in larger dimensions.

This talk is based on the works published in Refs. [1-3], in collaboration with C. Arenz, D. Burgarth, P. Facchi, V. Giovannetti, H. Nakazato, D. Orsucci, and S. Pascazio.

References

[1] D. K. Burgarth, P. Facchi, V. Giovannetti, H. Nakazato, S. Pascazio, and K. Yuasa, *Exponential Rise of Dynamical Complexity in Quantum Computing through Projections*, Nat. Commun. **5**, 5173 (2014).

[2] D. Orsucci, D. Burgarth, P. Facchi, H. Nakazato, S. Pascazio, K. Yuasa, and V. Giovannetti, *Hamiltonian Purification*, J. Math. Phys. **56**, 122104 (2015).

[3] C. Arenz, D. Burgarth, P. Facchi, V. Giovannetti, H. Nakazato, S. Pascazio, and K. Yuasa, *Universal Control Induced by Noise*, Phys. Rev. A **93**, 062308 (2016).

Pseudomode quantum jump and memory effect of non-Markovian dynamics

Yuta Ohyama[1] and Yasuhiro Tokura[2]
Graduate School of Pure and Applied Sciences, University of Tsukuba[1,2],
s1520221@u.tsukuba.ac.jp[1], tokura.yasuhiro.ft@u.tsukuba.ac.jp[2]

We study non-Markovian dynamics of a two level atom, using pseudomode method [1]. Pseudomode method is one of the methods treating non-Markovian dynamics. With this method, non-Markovian dynamics of the system of interest can be mapped to Markovian dynamics of a combined system of the system and pseudomodes. So we can apply methods for Markovian dynamics to the combined system. The purpose of this work is that we get a physically intuitive insight into the memory effect of non-Markovian dynamics.

The Hamiltonian for our system is

$$H = \frac{\hbar\omega_0}{2}\sigma_z + \sum_k \hbar\omega_k b_k^\dagger b_k + \sum_k \hbar g_k \left(\sigma_+ b_k + \sigma_- b_k^\dagger\right), \tag{1}$$

and the initial state of the reservoir is vacuum state. For simplicity, we consider that the density of states is described by a frequency dependent function $D(\omega)$ which is a sum of Lorentzians as follows

$$D(\omega) = \sum_{l=1}^{L} \frac{W_l \Gamma_l}{(\omega - \omega_l)^2 + (\Gamma_l/2)^2}, \tag{2}$$

and $\omega_l, W_l, \Gamma_l > 0$. With quantum jump approach [2], the probability density of a photon emission from the combined system is $p(t) = -\frac{d}{dt}\langle\tilde{\Psi}(t)|\tilde{\Psi}(t)\rangle$. Here $|\tilde{\Psi}(t)\rangle$ is a single quantum trajectory governed by a Schrödinger equation with a non-Hermitian Hamiltonian,

$$H_{\text{eff}} = \frac{\hbar\omega_0}{2}\sigma_z + \sum_{l=1}^{L}\left(\hbar\omega_l - i\frac{\hbar\Gamma_l}{2}\right)c_l^\dagger c_l + \sum_{l=1}^{L}\hbar\sqrt{W_l}\Omega\left(\sigma_+ c_l + \sigma_- c_l^\dagger\right), \tag{3}$$

where $c_l^\dagger$ and c_l are the bosonic creation and annihilation operators for the pseudomode labeled by l and Ω is a strength of the total coupling $\Omega^2 = \sum_k g_k^2$.

We discuss about the dynamics of $p(t)$ and the expectation value of jump time $\langle t\rangle$. The expectation value $\langle t\rangle$ can be divided into $\langle t\rangle_S$ and a sum of $\langle t\rangle_l$, which are the expected time length that states of the system and pseudomode l are in their excited state, respectively. In Markovian limit, $\langle t\rangle_l$ and $\langle t\rangle_S$ converge to 0 and the same value of Markovian dynamics of a two level atom, respectively. The Markovian approximation means that the reservoir has no memory so that we can infer that $\langle t\rangle_l$ reflects the memory effect of non-Markovian dynamics. In particular, we consider the damped Jaynes Cummings model, which is a model of a two level atom in a lossy cavity.

This work was supported by CREST, JST.

[1] B. M. Garraway, Phys. Rev. A 55, 2290 (1997).
[2] M. B. Plenio and P. L. Knight, Rev. Mod. Phys. 70, 101 (1998).

Solutions of the Schrödinger equation in terms of the Heun functions

Artur Ishkhanyan
Institute for Physical Research, NAS of Armenia, Ashtarak 0203, Armenia
aishkhanyan@gmail.com

We review the cases for which the Schrödinger equation is solved in terms of the general and confluent Heun functions. We present the possible choices for the coordinate transformation that provide energy-independent potentials that are proportional to an energy-independent continuous parameter and have a shape independent of that parameter. In contrast to the hypergeometric case, no Heun potential can in general be transformed into another one by specifications of the involved parameters.

We show that there exist in total 29 independent Heun potentials. There are eleven independent potentials that admit the solution in terms of the general Heun functions , for nine independent seven-parametric potentials the solution is given in terms of the confluent Heun functions , there are three independent double-confluent and five independent bi-confluent Heun potentials (the six-parametric Lamieux-Bose potentials), and one tri-confluent Heun potential (the general five-parametric quartic oscillator).

There are several independent potentials that present distinct generalizations of either a hypergeometric or a confluent hypergeometric classical potential, some potentials possess sub-cases of both hypergeometric types, and others possess particular conditionally integrable ordinary or confluent hypergeometric sub-potentials. We present several examples of explicit solutions for the latter potentials.

We show that there exist other exactly or conditionally integrable sub-potentials the solution for which is written in terms of simpler special functions. However, these are solutions of different structure. For instance, there are sub-potentials for which each of the two fundamental solutions of the Schrödinger equation is written in terms of irreducible combinations of hypergeometric functions . Several such potentials are derived with the use of deformed Heun equations . A complementary approach is the termination of the hypergeometric series expansions of the solutions of the Heun equations.

Rationally extended exactly solvable potentials and the potential algebra

Rajesh Kumar Yadav
Department of Physics, S. P. College, S. K. M. University, Dumka-814101 (India)
rajeshastrophysics@gmail.com

We discuss the rationally extended exactly solvable real and PT symmetric complex potentials whose bound state spectrums are in terms of exceptional Jacobi orthogonal polynomials. These potentials are isospectral to their conventional counterparts. We obtain these extended potentials by using potential algebra (or group theoretical) approach. The modified generators of the associated potential groups corresponding to these extended potentials are constructed by introducing a new operator $U(x, J_3 \pm \frac{1}{2})$. The Hamiltonian of these extended potentials are written in terms of the corresponding Casimir operators.

Spontaneous breaking of time-reversal symmetry in open quantum systems

Gonzalo Ordonez[1], Naomichi Hatano[2]
Physics and Astronomy Department, Butler University[1] *Institute of Industrial Science, University of Tokyo*[2],
gordonez@butler.edu[1], hatano@iis.u-tokyo.ac.jp[2]

We consider classes of open quantum systems modeled by a tight-binding Hamiltonian as well as by the Friedrichs model. We start with a time-reversal symmetric expansion of unity involving discrete eigenstates of the Hamiltonian. This expansion includes complex-conjugate pairs of resonant and anti-resonant states. We show how time-reversal symmetry is spontaneously broken as an initially time-reversal invariant state evolves in time. We show that there is a time-scale for the breaking of time-reversal symmetry, which we associate with the Zeno time. We also compare the time symmetric expansion with a time-asymmetric expansion used previously by several researchers including the first author. We show how the present time-symmetric expansion bypasses the non-Hilbert nature of the resonant and anti-resonant states, which previously introduced divergences into the time-asymmetric expansion.

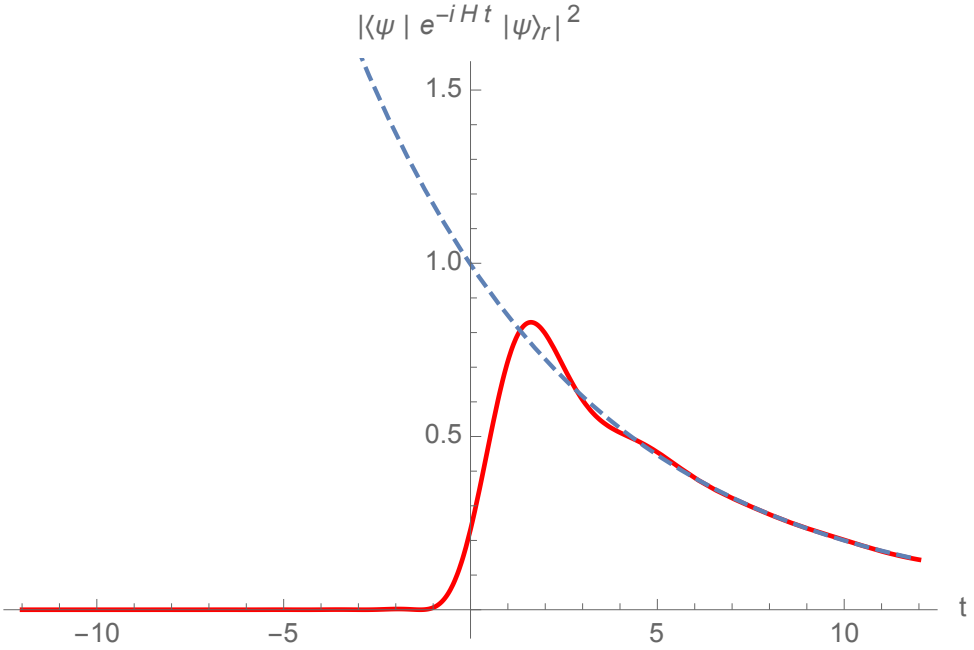


Figure 1: The resonant component of the survival probability in a tight-binding model (solid line) compared to the non-Hilbert state component (dashed line). The resonant component is spontaneously suppressed for $t < 0$ while the non-Hilbert state component diverges exponentially.

Complex Langevin approach to the sign problem

Shinji Shimasaki
Research and Education Center for Natural Sciences, Keio University,
Hiyoshi 4-1-1, Yokohama, Kanagawa 223-8521, Japan
shinji.shimasaki@keio.jp

The complex Langevin method (CLM) is a promising method for solving the sign problem in the path integral with a complex weight based on a Langevin equation for complexified dynamical variables.

Although the CLM has been known to have a restricted range of applicability, a recent progress of the method makes it possible to study various interesting systems, including finite density QCD in some parameter region.

In this talk, we will present the basic idea of the CLM as well as the recent development of the method. We first explain the condition for the CLM to work and then a new technique, called gauge cooling, which is inevitable for the application to lattice gauge theory. We also demonstrate the validity of the CLM with the gauge cooling in chiral random matrix theory and finite density QCD.

On the possible existence of 4n resonances

R. Lazauskas[1], J. Carbonell[2], E. Hiyama[3] and M. Kamimura[4]
IPHC, IN2P3-CNRS/Univ. Louis Pasteur BP 28, F-67037 Strasbourg Cedex 2, France[1], *Institut Physique Nucléaire, Univ. Paris-Sud, IN2P3-CNRS, 91406 Orsay Cedex, France*[2], *Nishina Center, RIKEN, Wako, 351-0198, Japan*[3], *Department of Physics, Kyushu University, Fukuoka 812-8581, Japan* [4]
rimantas.lazauskas@iphc.cnrs.fr[1]

We examine possibility for a four neutron system to posses a narrow resonance as suggested by a recent experimental result in RIKEN [1]. Since any sensitive modification of the nucleon-nucleon (NN) potentials or on the leading contributions of the three-nucelon (NNN) forces affect strongly the nuclear chart, we have introduced [2] a phenomenological T = 3/2 three neutron force, in addition to a realistic NN interaction, as an artefact to accommodate a 4n nearthreshold states. We inquired what would be the strength of such a 3n force in order to generate a resonance compatible with the experimental findings. The reliability of the resulting three-neutron force in the T = 3/2 channel is examined, by analyzing its consistency with the low-lying T = 1 states of ^{4}H, ^{4}He and ^{4}Li and the ^{3}H + n scattering.

Two independent configuration space methods are used in solving the four-body problem: the Gaussian expansion method [3, 4, 5] to solve the Schrodinger equation and the Lagrange-mesh technique applied to solve the Fadeev-Yakubowsky equation [6]. The boundary conditions related to the four-body problem in the continuum are implemented by using the complex scaling method [7, 8] and the position of the 4n resonances in the complex energy-plane are determined.

References

[1] K. Kisamori *et al.*, Phys. Rev. Lett. 116 (2016), 052501

[2] E. Hiyama, R. Lazauskas, J. Carbonell, N. Kamimura, Phys. Rev. C93 (2016), 044004

[3] M. Kamimura, Phys. Rev. A **38**, 621 (1988).

[4] H. Kameyama, M. Kamimura, and Y. Fukushima, Phys. Rev. C **40**, 974 (1989).

[5] E. Hiyama, Y. Kino, and M. Kamimura, Prog. Part. Nucl. Phys. **51**, 223 (2003).

[6] R. Lazauskas and J. Carbonell, Phys. Rev. C **72**, 034003 (2005), Phys. Rev. C **71**, 044004 (2005).

[7] Nuttal, J. and Cohen, H. L., Phys. Rev. **188** (1969) 1542.

[8] J. Carbonell, A. Deltuva, A.C. Fonseca, R. Lazauskas, Prog. Part. Nucl. Phys. **74**, 55 (2014).

CRCVM + ABC calculation for the three-body continuum

M. Iwasaki[1], R. Otani[1], M. Nakao[1], M. Ito[1], and M. Kamimura[2]
Department of Pure and Applied Physics, Kansai University, 3-3-35 Yamate-cho Suita, Osaka 564-8680, Japan[1], RIKEN Nishina Center for Accelerator-based Science, RIKEN, 2-1 Hirosawa, Wako, Saitama 351-0198, Japan [2]
itomk@kansai-u.ac.jp[1]

The absorbing boundary condition (ABC) method, which introduces the imaginary potential outside of a total system, is one of powerful methods to handle the continuum and resonant states in few-body systems [1, 2]. In the previous studies, there are several applications of ABC to the three-body problems, but no coordinate rearrangements are taken into account [1]. The inclusion of the rearrangement channels, which means the rearrangement of the Jacobi coordinate among the three particles, is called the coupled rearrangement channels variational method (CRCVM) [3]. The CRCVM is essential to obtain the rapid convergence of the total binding energy in a three-body system [3].

In the present study, we apply the ABC method to the three-body problem, which takes into account the rearrangement channels completely. We handle the identical three-boson system, which interacts by a simple Gaussian potential [1], and the S wave configurations are considered for the relative motion among the three bosons. The rearrangement channel is explicitly considered by imposing the boson symmetry among the identical three-bosons. The absorbing potential is placed among the pair of two bosons. As for the functional form of the absorber, the polynomial function is employed [1,2].

The eigenvalues obtained by the ABC method are distributed in the complex energy plane, and the resonant states are clearly separated from the continuum spectra. We have also calculated the strength function of the isoscalar monopole operator, which leads to the three-body isotropic breakup. The strength function is calculated by applying the extended completeness relation of the ABC solutions [2]. In the present report, we will demonstrate that the ABC method nicely works in the three-body calculation with the complete rearrangement channels.

Furthermore, we are now developing the ABC method in the full microscopic cluster model, generator coordinate method (GCM) [4]. The application of ABC to the two-body GCM calculation has already been successful [5] but the formulation of ABC + GCM in the three-body system still remains unclear. In the present report, we will discuss the formulation of ABC in the three-body GCM calculation.

[1] H. Masui and Y. K. Ho, Phys. Rev. C**65**, 054305 (2002).
[2] M. Iwasaki et al., Prog. Exp. Theor. Phys. **2015**, 023D01 (2015).
[3] E. Hiyama et al., Prog. Part. Nucl. Phys. **51**, 223 (2003).
[4] H. Horiuchi et al., Suppl. Prog. Theor. Phys. **62**, 90 (1977).
[5] M. Ito and K. Yabana, Prog. Theor. Phys. **113**, 1047 (2005).

Levels and widths statistics of the quantum many-body systems

Shoujirou Mizutori[1], Hirokazu Aiba[2]
Kansai University of Welfare Sciences[1], Kyoto Koka Women's College[2]
mizutori@tamateyama.ac.jp[1], h_aiba@mail.koka.ac.jp[2]

Recently, it is claimed that the widths of the low-lying neutron resonances do not follow the Porter-Thomas(PT) distribution, thus indicate the failure of the random matrix theory[1]. Against this claim, a possibility is pointed out that the super-radiant phenomena, namely the width collectivization due to the strong coupling to the contituum, lead to the modification of the statistics[2]. Another possibility is proposed that the correlation between the target nuclei and the compound one (the target nuclei plus a neutron) causes the deviation of the statistics of the width distribution[3]. While these considerations focus on the widths only, it is also interesting to examine the statistics of the levels as well to judge whether these explanations hold.

We adopt the two-body random ensemble as the intrinsic Hamiltonian, and construct non-Hermite effective Hamiltonian with the imaginary part due to the coupling to the continuum evaluated consistently with the intrinsic Hamiltonian. With this model, we analyze the nearest neighbor level spacings(NNLS), fitting with the Brody distribution, and the width statistics fitting with the χ^2 function. Changing the strength κ of the couplings to the continuum, we try to clarify the condition in which the width distribution deviates from the PT distribution(for which the degree of freedom ν of the χ^2 function is 1.0) , and how the parameter β of the Brody distribution deviates from 1.0 (the value for the Wigner distribution) when irregularity is observed in the width distribution.

It turns out that the couplings between the intrinsic states and the continuum depend strongly on the energy. Therefore, we do not analyze whole spectrum but concentrate on a narrow region in which the energy dependence of the couplings can be neglected. We choose the region around the expectation value of the Hamiltonian with target + one particle state. Then, if the κ is very small, the widths follows the PT distribution while the NNLS follows the Wigner distribution. With increasing κ, the widths deviate from the PT distribution and the NNLS deviate from the Wigner distribution, simultaneously. Further increase of the coupling restore the PT distribution of the widths and the Wigner distribution of the NNLS, respectively.

The shifts of the positions of the poles in the energy-width plane indicate that these simultaneous deviations are inevitable. Namely, if the deviation of the distribution of the neutron width from the PT one would be because of the width collectivization due to the strong coupling to the continuum, one would also observe the deviation of the NNLS distribution.

[1] P.E. Koehler et al., Fortschr. Phys. **61**, 80-94 (2013)
[2] G.L. Celardo, N. Auerbach, F.M. Izrailev, and V.G. Zelevinsky, Phys. Rev. Lett. **106**, 042501(2011)
[3] A. Volya, Phys. Rev. C **83**, 044312 (2011)

Single-neutron resonance and pairing correlation in neutron-rich nuclei

Yoshihiko Kobayashi[1], Masayuki Matsuo[2]
Graduate School of Science and Technology, Niigata University, Japan[1]*, Department of Physics, Faculty of Science, Niigata University, Niigata, Japan*[2]
kobayashi@nt.sc.niigata-u.ac.jp[1], matsuo@nt.sc.niigata-u.ac.jp[2]

Almost all nuclei except for closed shell nuclei have superfluidity which is originated from pairing correlation. In weakly bound nuclei such as neutron-rich nuclei, the pairing correlation causes configuration mixing involving both bound and unbound (continuum) single-particle orbits since the Fermi energy locates near continuum.

This continuum coupling brings about novel features. An interesting example is the possible manifestation of a new type of resonance generated by the pairing correlation and the continuum coupling, called the quasi-particle resonance [1,2]. If one describes a single-particle scattering problem within the scheme of Bogoliubov's quasi-particle theory, even a scattering state becomes a quasi-particle state that has both "particle" and "hole" components. In other words, An unbound particle couples a Cooper pair and a hole orbit, then forms the quasi-particle resonance.

The Hartree-Fock-Bogoliubov theory in coordinate space can describe such weakly bound superfluid system wthin the scattering wave functions [3,4]. It is a mean-field theory including pair condensate field. The quasi-particle resonance is predicted by this theory.

We have focused on the effects of pairing correlation on low-lying p wave quasi-particle resonance in present study. The phase shifts, elastic cross sections, resonance energy, and resonance width are calculated. We have investigated their dependence on the pairing correlation. Through this investigation, we have discovered that the pairing correlation has the effect of reducing the resonance width [5].

1. S. T. Belyaev, A. V. Smirnov, S. V. Tolokonnikov, and S. A. Fayans, Sov. J. Nucl. Phys. 45, 783 (1987).
2. A. Bulgac, arXiv:nucl-th/9907088 (1980).
3. J. Dobaczewski, H. Flocard, and J. Treiner, Nucl. Phys. A 422, 103 (1984).
4. J. Dobaczewski, W. Nazarewicz, T. R. Werner, J. F. Berger, C. R. Chinn, and J. Dechargé, Phys. Rev. C 53, 2809 (1996).
5. Y. Kobayashi and M. Matsuo, Prog. Theor. Exp. Phys. 013D01 (2016).

Atomic and Molecular Siegert States in a strong electric field

Toru Morishita
Institute for Advanced Science, The University of Electro-Communications
toru@pc.uec.ac.jp

The ionization of atoms and molecules by a static electric field is one of the fundamental problems in quantum mechanics. A new wave of interest in this problem over the past two decades has been motivated by the appearance of intense low-frequency laser pulses. The ionization of atoms and molecules by laser pulses is the first step for a variety of strong-field-induced rescattering phenomena of current interest, such as the generation of high-order harmonics and high-energy photoelectrons. An accurate quantitative description of this step is obviously required. Recently, we have established an efficient theoretical and computational method for calculating Siegert states in a static electric field, which are the solutions of the stationary Schrödinger equation satisfying the regularity and outgoing-wave boundary conditions, based on the adiabatic expansion in parabolic coordinates. This makes it possible to calculate the Siegert eigenvalue $E = E - i\Gamma/2$ defining the energy E and ionization rate Γ as well as the transverse momentum distribution of ionizing electrons with respect to the field direction. In this work we present the basics of the Siegert state method and some illustrative calculations of the Siegert states for some atomic and molecular systems. We also discuss atomic and molecular dynamics, including chemical reactions via an exceptional point of Siegert states.

Boundary Conditions of Resonant states.

G.A. Luna-Acosta[1], A.A. Fernández-Marín[2], Charles Poli[3]
Instituto de Física, Benemérita Universidad Autónoma de Puebla, Apartado Postal J-48, Puebla 72570, Mexico[1], *Instituto de Ciencias Físicas, Universidad Nacional Autónoma de México, A.P. 48-3, 62251 Cuernavaca, Morelos, Mexico*[2]
glunaacosta@gmail.com[1], antoniofdzm@gmail.com[2], cpoli83@hotmail.fr[3]

We analyze the resonant behavior of one-channel one dimensional systems with a finite range of interaction. This is done by studying various scattering functions (time delay, probability trapping and cross section) within the framework of Reaction Matrix theory and also in terms of an effective non-hermitean Hamiltonian H_{eff}. It is shown that in order to use a one (reaction matrix) level approximation the appropriate or physical boundary condition of the resonant state must be used. We show a simple procedure to determined the physical boundary conditions of the resonant states. It is also shown that only for nearly Neumann boundary conditions the cross section attains the Breit-Wigner form. In the opposite extreme of resonant states obeying nearly Dirichlet boundary conditions, the cross section attains becomes an inverted Breit-Wigner shape. These results naturally lead us to question the definition of resonance in terms of the scattering cross section.

Resonances and the Complex Absorbing Potential method

M. Melgaard
School of Mathematical and Physical Sciences, University of Sussex, Great Britain
m.melgaard@sussex.ac.uk

The Complex Absorbing Potential (CAP) method is widely used to approximate resonances, both for nonrelativistic and relativistic Hamiltonians. We provide an introduction to the method and, in the semiclassical limit $\hbar \to 0$ we consider resonances near the real axis and we establish the CAP method rigorously for the perturbed Dirac operator by proving that individual resonances are perturbed eigenvalues of the non-Hermitian CAP Hamiltonian, and vice versa.

Scattering from two-piece rising potentials: a new avenue of resonances

Sachin Kumar
Theoretical Physics Section, Bhabha Atomic Research Centre, Trombay, Mumbai, 400085, India
sachinv@barc.gov.in

We study scattering from potentials that rise monotonically on one side; this is generally avoided. We report that resonant states are absent in such potentials when they are smooth and single-piece (like in the cases of Morse oscillator, exponential and linear potentials). But when these potentials are made two-piece, resonances can occur. We further show that rising potentials next to a well/step/barrier are rich models of multiple resonances (Gamow's decaying states) in one- dimension. We use linear, parabolic and exponential profiles as rising part and find complex-energy poles, $\mathcal{E}_n = E_n - i\Gamma_n/2$ ($\Gamma_n > 0$), in the reflection amplitude (s-matrix). The appearance of peaks in Wigner's (reflection) time-delay at $E = \epsilon_n$ (close to E_n) and spatial catastrophe in the eigenfunction confirm the existence of resonances and meta-stable states in these systems.

Irreversibility and complex spectral analysis of Liouvillian and Hamiltonian dynamics in terms of non-Hermitian operators

Tomio Petrosky
Center for Studies in Statistical Mechanics and Complex Systems, The University of Texas at Austin
petrosky@physics.utexas.edu

General overview review of recent results of the complex spectral analysis of the Liouvillian dynamics as well as the Hamiltonian dynamics developed by Austin-Osaka group is presented. From the microscopic fundamental laws of physics, irreversible processes can be derived through the resonance singularities in the so-called small-denominator for the open systems without relying upon any phenomenological arguments such as the coarse graining approximation. Due to the resonance singularities, the Hermitian generator of motion in the Hilbert space leads to non-Hermitian effective Liouvillian and/or Hamiltonian with complex eigenvalues in the extended function space. These effective operators share the same eigenvalue with the original Liouvillian and/or Hamiltonian. The imaginary parts of the eigenvalues give the transport coefficients in irreversible processes. In this talk the irreversible process associated with the Jordan block that has no counter part in Hermitian dynamics will be discussed.

Microscopic description of irreversible processes in quantum Lorentz gas with resonance state of the Liouvillian

Kazunari Hashimoto
Graduate School of Interdisciplinary Research, University of Yamanashi, Kofu 400-8511, Japan
hashimotok@yamanashi.ac.jp

We investigate irreversible processes in a weakly-coupled one-dimensional quantum perfect Lorentz gas by means of the resonance state of the Liouville-von Neumann operator (Liouvillian). The resonance state of the Liouvillian is constructed by solving the non-linear eigenvalue problem of the effective Liouvillian, which is obtained by extending the well-known Brillouin-Wigner-Feshbach formalism to the Liouville-von Neumann equation. We solve the non-linear eigenvalue problem without making any phenomenological operations, such as a coarse-graining of space-time, or the molecular chaos. As a result, we obtain irreversible processes in a purely dynamical basis in all space and time scale including the microscopic atomic interaction range that is much smaller than the mean-free length. In this talk, we discuss a limitation of the usual phenomenological Boltzmann equation, as well as an extension of the Boltzmann equation to entire space-time scale based on the solution.

Properties of exceptional points in open quantum systems

Savannah Garmon
Department of Physical Science, Osaka Prefecture University
sgarmon@p.s.osakafu-u.ac.jp

We demonstrate some general properties of open quantum systems in the vicinity of an exceptional point, at which two or more eigenstates coalesce and the usual diagonalization scheme of the Hamiltonian fails. In the literature this scenario is often treated by relying on a heuristic effective Hamiltonian; however, here we treat the problem analytically relying on a generalized perturbative method using simple models as examples. We show that the exceptional points are mathematically equivalent to branch points in the parameter space of the Hamiltonian, which is illustrated in the non-analytic properties of the eigenvalues in the vicinity of the exceptional points. As a direct result of these eigenvalue properties, the norm of the associated eigenstates diverges at the exceptional point. Using our analytic approach, we show that the Hamiltonian can only be reduced to Jordan block form at the exceptional point [1]. Finally we briefly discuss the influence of an exceptional point on the evolution of an initially prepared state, emphasizing that continuum threshold effects [2] may play a significant role.

[1] G. Bhamathi and E. C. G. Sudarshan, Int. J. Mod. Phys. B **10**, 1531 (1996).
[2] S. Garmon, T. Petrosky, L. Simine, and D. Segal, Fortschr. Phys. **61**, 261 (2013).
[3] S. Garmon, I. Rotter, N. Hatano, and D. Segal, Int. J. Theor. Phys. **51**, 3536 (2012).
[4] S. Garmon, M. Gianfreda, and N. Hatano, Phys. Rev. A **92**, 022125 (2015).
[5] N. Hatano and G. Ordonez, J. Math. Phys. **55**, 122106 (2014).

Higher-order Time-Symmetry-Breaking Phase Transition due to meeting of an Exceptional Point and Fano Resonance

Satoshi Tanaka
Department of Physical Science, Osaka Prefecture University
stanaka@p.s.osakafu-u.ac.jp

We have theoretically investigated the time-symmetry breaking phase transition process for two discrete states coupled with a one-dimensional continuum by solving the nonlinear eigenvalue problem for the effective Hamiltonian associated with the discrete spectrum. We obtain the effective Hamiltonian with use of the Feshbach-Brillouin-Wigner projection method. Strong energy dependence of the self-energy appearing in the effective Hamiltonian plays a key role in the time-symmetry breaking phase transition: as a result of competition in the decay process between the Van Hove singularity and the Fano resonance, the phase transition becomes a higher-order transition when both the two discrete states are located near the continuum threshold.

In this talk, we also shed a new light on the nonlinear eigenvalue problem of the effective Hamiltonian in terms of the stability of the solutions. The time-symmetry breaking is regarded as a bifurcation in the complex plane of the solutions of the dispersion equation. According to the bifurcation theory, we define a *velocity field* toward the fixed point in the complex plane, and found out that a peculiar spiral flow appears around the resonance and anti-resonance states. We found out that the appearance of the spiral flow corresponds to the complex normalization factor of the eigenvector in the extended Hilbert space.

Decay dynamics in a periodically driven open quantum system

Kenichi Noba[1], Savannah Garmon[2], Hidemasa Yamane[2], Satoshi Tanaka[2], Tomio Petrosky[3]
Department of Mathematical Sciences, Osaka Prefecture University[1],
Department of Physical Science, Osaka Prefecture University[2],
Center for Complex Quantum Systems, The University of Texas at Austin[3]
noba@ms.osakafu-u.ac.jp[1]

The exponential and non-exponential decay processes of an open quantum system driven by an external oscillating field are investigated. We consider a system in which a discrete impurity is weakly coupled to a one-dimensional continuum and the energy of the impurity is driven by an oscillating field. The survival probability at the impurity is calculated in terms of the complex spectral analysis of the Floquet Hamiltonian. With this method we can unambiguously decompose the survival probability into the exponential decay component, the oscillating component, and the power law decay component. We discuss the effects of the driving field on the time evolution of these components.

The exponential component that dominates the survival probability in the intermediate time region is associated with the resonance state. The exponential decay rate given by the imaginary part of the eigenvalue is significantly modified by the driving field. In the small band case where the width of the continuum is small compared with the frequency of the driving field, the exponential decay is completely suppressed when the ratio between the amplitude and frequency of the driving field satisfies the zeroes of the Bessel function [1]. The oscillating component associated with bound states near the band edges does not decay because the eigenvalues are real. However, the amplitude of this component depends on the driving field and even vanishes when the corresponding eigenvalues move to the second Riemann sheet in the complex energy plane due to the influence of the driving field [2]. The power law decay component coming from the contribution of the continuum states dominates the survival probability in the long time region. During the intermediate time domain (long time near zone) the dynamics follows a t^{-1} decay, while in the asymptotic domain (long time far zone) it follows a t^{-3} decay [3]. The characteristic time scale dividing these two zones can be modified by the driving field [4].

[1] N. Yamada, K. Noba, S. Tanaka, T. Petrosky, Phys. Rev. B **86,** 014302 (2012).
[2] K. Noba, N. Yamada, Y. Uesaka, S. Tanaka, T. Petrosky, J. Phys. A: Math. Theor. **47,** 385302 (2014).
[3] S. Garmon, T. Petrosky, L. Simine, D. Segal, Fortschr. Phys. **61,** 261 (2013).
[4] N. Yamada, S. Garmon, H. Yamane, S. Tanaka, K. Noba, T. Petrosky, to be submitted.

Decay of two repulsively interacting particles

Sungyun Kim[1], Joachim Brand[2]
Institute of NT.IT fusion technology, Ajou University[1]*, Centre for Theoretical Chemistry and Physics, New Zealand Institute for Advanced Study, Massey University*[2]
rdecay@gmail.com[1],J.Brand@massey.ac.nz[2]

We study the decay of two repulsively interacting bosons tunneling through a delta potential barrier by direct numerical solution of the time-dependent Schrödinger equation. The solutions are analyzed according to the regions of particle presence: both particles inside the trap (in-in), one particle in and one particle out (in-out), and both particles outside (out-out). It is shown that the in-in probability is dominated by exponential decay, and its decay rate is predicted very well from outgoing boundary conditions. Up to a certain ranges of interaction strength the decay of in-out probability is dominated by the single particle decay mode. The decay mechanisms are adequately described by simple models.

Resonance on essential kaonic nuclear systems

Akinobu Dote[1], Takashi Inoue[2], Takayuki Myo[3]
KEK Theory Center[1], Nihon University, College of Bioresource Sciences[2], General Education, Faculty of Engineering, Osaka Institute of Technology[3]
dote@post.kek.jp[1], myo@rcnp.osaka-u.ac.jp[2], inoue.takashi@nihon-u.ac.jp[3]

In strangeness nuclear physics and hadron physics, nuclear systems with anti-kaons (kaonic nuclei, $\bar{K}$ nuclei) have been one of hot issues, because of their expected exotic nature. An anti-kaon ($\bar{K}$ meson = K^- and $\bar{K}^0$) is a pseudo-scalar meson involving a strange quark, and the attraction between an anti-kaon and a nucleon is considered to be so attractive that they can form a quasi-bound state which corresponds to a hyperon resonance $\Lambda(1405)$ existing at 27 MeV below the $\bar{K}N$ threshold. Due to such a strong $\bar{K}N$ attraction, an anti-kaon pulls nucleons close to itself when an anti-kaon is put into a nucleus. The anti-kaon behaves as if it were a seed of attraction, and is expected to form a dense state in a nucleus against the well-known nuclear incompressibility. Actually, an early study reported that in light kaonic nuclei could be dense and their density could amount to $2 \sim 4\rho_0$, where ρ_0 means a normal nuclear density, 0.16 fm^{-3} [1].

To clarify the exotic nature of kaonic nuclei, many theorists and experimentalists have eagerly investigated essential two systems: an excited hyperon $\Lambda(1405)$ which is a two-body $\bar{K}N$ quasi-bound system, and a three-body "K^-pp" system (composing two protons and a K^- meson, naively) which would be a prototype of kaonic nuclei. From theoretical point of view, the treatment of 1. coupled-channel problem and 2. resonance is important to investigate these systems, since the $\bar{K}N$ pair couples strongly to πY pair and these systems are not purely a bound state but a resonance. (Y means hyperons of Λ and Σ.) For instance, $\Lambda(1405)$ is certainly located energetically below $\bar{K}N$ threshold, but is above the $\pi\Sigma$ threshold and decays to the $\pi\Sigma$. In other words, $\Lambda(1405)$ is a so-called Feshbach resonance.

By the way, the complex scaling method is known to be a powerful tool to study resonances in the ordinary nuclear physics, because of great success in studies of resonant states of stable/unstable nuclei. Therefore, in our study of kaonic nuclei, we employ a coupled-channel complex scaling method (ccCSM) which can deal with the above-mentioned two ingredients simultaneously. In the present talk, I will introduce interests of kaonic nuclei from the viewpoint of physics, and report our comprehensive studies of $\Lambda(1405)$ and "K^-pp" with the ccCSM, based on our recent papers [2-4]. In addition, I would like to mention to the experiments on these systems performed and on-going at J-PARC.

[1] A. Doté, H. Horiuchi, Y. Akaishi and T. Yamazaki, Phys. Rev. C **70**, 044313 (2004).
[2] A. Doté, T. Inoue and T. Myo: Nucl. Phys. A **912** (2013) 66.
[3] A. Doté and T. Myo: Nucl. Phys. A **930** (2014) 86.
[4] A. Doté, T. Inoue and T. Myo: Prog. Theor. Exp. Phys. **2015** (2015) 043D02.

Two-body wave functions and compositeness from scattering amplitudes

Takayasu Sekihara
Advanced Science Research Center, Japan Atomic Energy Agency
sekihara@post.j-parc.jp

For a general two-body bound state in quantum mechanics, both in the stable and decaying cases, we establish a way to extract its two-body wave function in momentum space from the scattering amplitude of the constituent two particles.

For this purpose, we first show that the two-body wave function of the bound state corresponds to the residue of the off-shell scattering amplitude at the bound state pole, by considering solutions of both the Schrödinger and Lippmann–Schwinger equations. This means that solving the Lippmann–Schwinger equation at the bound state pole is equivalent to evaluating the two-body wave function of the bound state.

Then, we examine our scheme in several schematic models and extract the two-body wave function from the scattering amplitudes. We consider both stable and unstable bound states, and in single-channel and coupled-channels problems. With the schematic models we discuss general properties of the two-body wave function and compositeness for bound states. Of special interest is that we obtain the normalized two-body wave function of the bound state from the scattering amplitude, regardless of whether the bound state is stable or not; the norm of the two-body wave function, to which we refer as the compositeness, is unity for an energy independent interaction, while the compositeness deviates from unity for an energy dependent interaction, which can be interpreted to implement missing channel contributions.

As applications, we investigate the compositeness of several hadronic resonances. With our scheme, we may able to distinguish hadronic molecules, which are composed of two (or more) hadrons themselves, like deuteron as a proton–neutron bound state, rather than compact qqq states for baryons and $q\bar{q}$ for mesons. We evaluate the compositeness for candidates of hadronic molecules such as $\Lambda(1405)$ from hadron–hadron scattering amplitudes in hadron effective models.

Degenerate two-body and three-body coupled-channels system and the S-matrix pole behavior near the thresholds

Atsunari Konishi[1], Osamu Morimatsu[1,2,3], Shigehiro Yasui[4]
High energy accelerator research organization (kek)[1],
The university of Tokyo[2],
The graduate university for advanced study[3],
Tokyo institute of technology[4]
atsunak@post.kek.jp[1], osamu.morimatsu@kek.jp[2], yasuis@th.phys.titech.ac.jp[3]

Motivated by the existence of candidates for the exotic hadron whose masses are close to both of two-body and three-body hadronic thresholds, we investigated the S-matrix pole behavior near the thresholds in a degenerate two-body and three-body coupled-channels system.

Two-body and three-body coupled-channels scattering equations are formulated as effective three-body scattering equations in which effects induced by a coupling to the two-body channel are embedded as effective interactions in the three-body channel constructed by the Feshbach projection method. We solve the eigenvalue equations of the kernel of the scattering equations instead of the scattering equations themselves to obtain the S-matrix pole. Although the transition amplitudes have physical singularities, the kernel of the scattering equations does not, i.e. it has unphysical singularities. However, we show that this unphysical singularity problem can be resolved by an appropriate reorganization of the scattering equations and the mass renormalization.

The S-matrix pole behavior near the thresholds in the degenerate two-body and three-body coupled-channels system has universal property whose pole energy E is determined by an equation $g - E\log(-E) = 0$, that is, it is determined by one parameter g. The behavior is neither those of a single-channel two-body nor a three-body system. We expect the behavior is realized also for those candidates for the exotic hadrons that lie around the energy regions where two-body as well as three-body thresholds rest close to each other and play a key role in understanding those resonances.

Analysis of the $\Lambda(1405)$ resonance by energy-dependent complex $\bar{K}N$ potential

Kenta Miyahara[1], Tetsuo Hyodo[2]
Kyoto University[1]*, Yukawa Institute for Theoretical Physics*[2]
miyahara.kenta.62r@st.kyoto-u.ac.jp[1], hyodo@yukawa.kyoto-u.ac.jp[2]

The $\Lambda(1405)$ baryon resonance has been a hot topic for its several peculiar natures. The first is the difference from the usual three-body quark model. In contrast to ordinary hadrons, the $\Lambda(1405)$ is considered as the hadronic molecular state with an antikaon $\bar{K}$ and a nucleon N. The second peculiar nature is the so-called double pole structure, which indicate that the $\Lambda(1405)$ is expressed by the two resonance poles derived from the $\bar{K}N$ and $\pi\Sigma$ attractive interactions. In this work, to analyze the $\Lambda(1405)$ quantitatively, we construct the realistic $\bar{K}N$ local potential, and estimate the spatial structure of the $\Lambda(1405)$.

Our potential construction procedure is based on chiral unitary approach, which respects the chiral SU(3) symmetry in QCD. The $\bar{K}N$ local potential is constructed to reproduce the scattering amplitude. Here, we establish the method to accurately reproduce the amplitude even in the complex energy plane. Because the resonance pole lies in the complex plane, such construction procedure is necessary for the $\Lambda(1405)$ analysis. Furthermore, we consider the recent experimental data by the SIDDHARTA Collaboration for the first time. This precise data can significantly reduce the theoretical uncertainty, and is necessary to construct the realistic potential for the quantitative analysis.

The $\bar{K}N$ local potential is complex and energy dependent. To satisfy the essential conditions for quantum mechanics like the orthogonality condition, a careful treatment is necessary for a system with an energy-dependent potential. In the case of a real and energy-dependent potential, the treatment has been established respecting the continuity equation. We extend the treatment to the complex and energy-dependent potential with the Gamow vector. However, it is known that the physical value becomes complex with the Gamow vector, and it is difficult to extract the physical meaning from such complex value. In this work, we consider the root-mean distance of the $\bar{K}N$ system to analyze the spatial structure of the $\Lambda(1405)$, and find the way to extract the real value as the distance paying attention to the dump of the wave function. As a result, we find that the $\bar{K}N$ distance is relatively larger (~ 1.4 fm) than the typical hadronic scale (~ 0.8 fm) and the hadronic molecular picture of the $\Lambda(1405)$ is reasonable.

The realistic $\bar{K}N$ local potential is applicable to the calculation of the $\bar{K}$-nuclei, which may be qualitatively different from the usual nuclei because of the strong $\bar{K}N$ interaction. In the few-body calculation, the treatment with the complex and energy-dependent potential is important, and should be established based on the above way for the two-body calculation.

Quantum systems with complex energy spectra

Miloslav Znojil
Nuclear Physics Institute of the Czech Academy of Sciences, 250 68 Řež, Czech Republic
znojil@ujf.cas.cz

Dynamical aspects of quantum systems are often controlled by external forces. Two classes of toy models of such a mechanism will be presented. The first one will be represented by a non-Hermitian equilateral q-pointed star-shaped quantum graph in which the effect of the external forces is mimicked by the rotation-symmetric complex Robin boundary conditions at the outer ends of the wedges. In our second class of models we consider more complicated "thick" graphs *alias* thin wave guides in more dimensions while we simplify the simulation of the effect of external forces. We will pay attention to the dynamical regime in which the discrete spectrum ceases to be real. In certain special cases, for illustration, closed asymptotic-expansion formulae will be derived.

.

Prepared partially in collaboration with Denis Borisov.

Complex normalization constant of eigenstates for a non-Hermitian Hamiltonian and Fano profile in the intra-atomic photo absorption spectrum

Taku Fukuta[1], Savannah Garmon[1], Tomio Petrosky[2,3], Satoshi Tanaka[1]
Department of Physical Science, Osaka Prefecture University[1],
Center for Complex Quantum Systems, The University of Texas at Austin[2],
Institute of Science Industry, University of Toyko[3]
s_t.fukuta@p.s.osakafu-u.ac.jp[1]

The intra-atomic photo absorption spectrum of an impurity atom embedded in a semi-infinite quantum wire is investigated in terms of the complex eigenvalue problem for an effective non-Hermitian Hamiltonian describing an open quantum system[1,2]. The effective Hamiltonian for the impurity state is constructed from the original Hermitian Hamiltonian following the Brillouin-Wigner-Feshbach projection method. Bi-orthonormal and bi-complete eigenstates with complex eigenvalues of the total Hamiltonian are obtained in the dual space, which is a complex extension of the usual Hilbert space. In the dual space, the normalization constant of the eigenstate is in general a complex number. The imaginary part of the normalization constant manifests itself as a non-symmetric profile (the so-called Fano profile) appearing in the photo absorption spectrum, as it leads to a principal-part-type contribution. Our treatment of the Fano profile in terms of the complex spectral representation reveals a new aspect of the Fano resonance, which may also appear in other optical spectra, such as resonance Raman spectrum, emission spectrum, and so on.

[1] E. C. G. Sudarshan, C. B. Chiu, and Vittorio Gorini, Phys. Rev. D **18,** 2914 (1978).
[2] T. Petrosky, I. Prigogine and S. Tasaki, Physica A **173,** 175 (1991).

The dynamical Casimir effect of a metastable excited atom driven by a time periodic external field

Hidemasa Yamane[1,*], Ken-ichi Noba[2], Tomio Petrosky[3,4], and Satoshi Tanaka[1]
Department of Physical Science, Osaka Prefecture University[1],
Department of Mathematical Science, Osaka Prefecture University[2],
Center for Complex Quantum Systems, The University of Texas at Austin[3],
Institute of Industrial Science, University of Tokyo[4]
s_h.yamane@p.s.osakafu-u.ac.jp*

We have studied the dynamical Casimir effect of a metastable excited atom whose energy is driven by a periodic external field in terms of the complex spectral representation of the Floquet Hamiltonian. The non-Hermitian effective Hamiltonian for the impurity state is constructed from the original Hermitian Hamiltonian in terms of the Brillouin-Wigner-Feshbach projection method. We have revealed characteristically different types of a photon emission from the atom. We have found that a virtual photon initially localized at the atom is emitted as a Zeno photon in a short time scale due to the branch point contribution associated with the initial dressed field. In addition, a new virtual cloud emerges around the atom due to the induced branch point effects by the external field. Since the periodic external field yields many Floquet bands, this new virtual cloud is formed as a superposition of many branch point contributions, resulting in a unique space-time evolution of the photon emission in the dynamical Casimir effect.

Besides the branch point effect, Floquet resonance states gives a delocalized photon emission with an exponential intensity growth in space. As a result of the interference between many Floquet resonance states, we have found out a pulsed photon emission in space.

We also show that the spectral profiles of the photon emission due to the branch point effect and the resonance state effect are characteristically different and that the latter effect is responsible for the high harmonic generation in dynamical Casimir effect.

Landau-Zener transitions and Dykhne formula in simple continuum models

Yujin Dunham[1], Savannah Garmon[2], Gonzalo Ordonez[3]
Department of Physical Sciences, Osaka Prefecture University[1,2]*, Department of Physics and Astronomy, Butler University*[3]
s_y.dunham@p.s.osakafu-u.ac.jp[1], sgarmon@p.s.osakafu-u.ac.jp[2]

The Landau-Zener model describing the interaction between two linearly driven discrete levels is useful in describing many simple dynamical systems; however, no system is completely isolated from the surrounding environment. Here we examine two generalizations of the original Landau-Zener model to study simple environmental influences. First we consider a model in which one of the discrete levels is replaced with a continuum, in which we find that the survival probability for the initially occupied diabatic level is unaffected by the presence of the continuum. This result can be predicted by assuming that each step in the evolution for the diabatic state evolves independently according to the Landau-Zener formula, even in the continuum limit. We also show that, at least for the simplest model, this result can also be predicted with the natural generalization of the Dykhne formula for open systems [1]. We also consider a second generalization in which a continuum is added to the traditional Landau-Zener model. Here we find that there is a shift in the survival probability due to the continuum [2]. We also observe dissipation as the non-escape probability from the discrete levels is no longer equal to one.

[1] A. M. Dykhne, Sov. Phys. JETP 14, 941 (1962).
[2] A. Dodin, S. Garmon, L. Simine, and D. Segal, J. Chem. Phys. **140**, 124709 (2014).

Generalized KdV equation induced from position-dependent effective mass quantum models and mass-deformed soliton solution through inverse scattering transform

Asish Ganguly
Department of Mathematics, Indian Institute of Technology Kharagpur, Kharagpur 721302, India
aganguly@maths.iitkgp.ernet.in, gangulyasish@rediffmail.com

We consider one-dimensional stationary position-dependent effective mass quantum model and derive a generalized KdV equation in (1+1) dimension through Lax pair formulation, one being the effective mass Schroedinger operator and the other being the time-evolution of wave functions. We obtain an infinite number of conserved quantities for the generated nonlinear equation and explicitly show that the new generalized KdV equation is an integrable system. Inverse scattering transform method is applied to obtain general solution of the nonlinear equation, and then N-soliton solution is derived for reflectionless potentials. Finally, a special choice has been made for the variable mass function to get mass-deformed soliton solution. The influence of position and time-dependence of mass and also of the different representations of kinetic energy operator on the nature of such solitons is investigated in detail. The remarkable features of such solitons are demonstrated in several interesting figures and are contrasted with the conventional KdV-soliton associated with constant-mass quantum model.