Non-Hermitian quantum mechanics and localization

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Collaborators

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N.H. and D.R. Nelson:

PRL 77, 570 (1996)

PRB 56, 8651 (1997)

PRB 58, 8384 (1998)

N.H.:

Physica A 254, 317 (1998)

Why non-Hermitian?

- provides a new viewpoint of Hermitian physics
- expresses physical phenomena effectively

(1) Anderson localization

Localization length

(2) Resonant states

Resonance lifetime

(3) Flux-line pinning in type-II superconductors

Vortex depinning transition

Outline

1. Non-Hermitian Anderson model and delocalization
2. Motivation I —— Localization length
3. How does the delocalization happen?
4. Numerical examples
5. Motivation II —— Resonant states
6. Motivation III —— Vortex depinning in type-II superconductors

Non-Hermitian Anderson model

Continuum model

$$H = \frac{(p + ig)^2}{2m} + V(x)$$

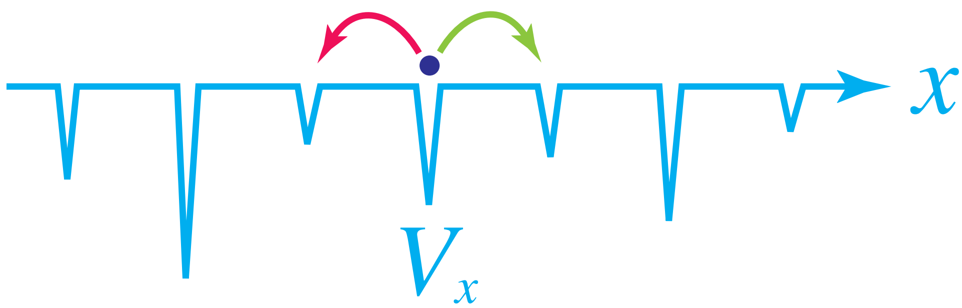
g: "imaginary vector potential" (constant)

$V(x)$: random scalar potential

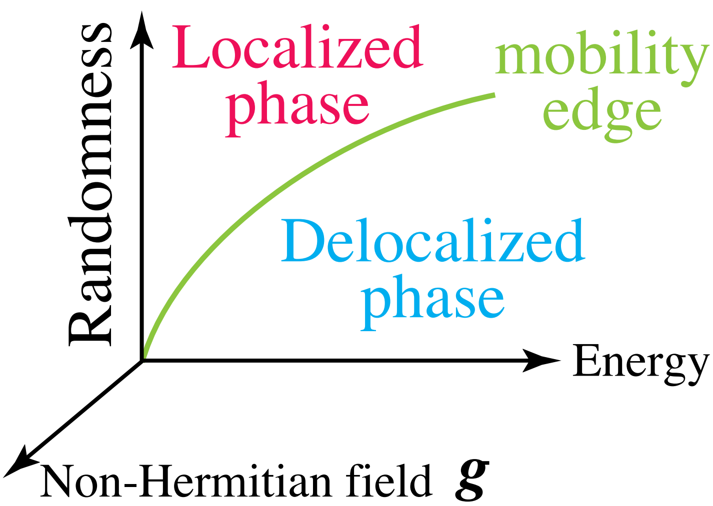
$g=0$: (conventional) 1-electron Anderson model

Lattice model

$$H = -\frac{t}{2}\sum_{\boldsymbol{x}}\sum_{\nu=1}^{d}\left(e^{\bar{\boldsymbol{g}}\cdot\boldsymbol{e}_\nu}\left|\boldsymbol{x}+\boldsymbol{e}_\nu\right\rangle\left\langle\boldsymbol{x}\right| + e^{-\bar{\boldsymbol{g}}\cdot\boldsymbol{e}_\nu}\left|\boldsymbol{x}-\boldsymbol{e}_\nu\right\rangle\left\langle\boldsymbol{x}\right|\right) + \sum_{\boldsymbol{x}} V_{\boldsymbol{x}}\left|\boldsymbol{x}\right\rangle\left\langle\boldsymbol{x}\right|$$



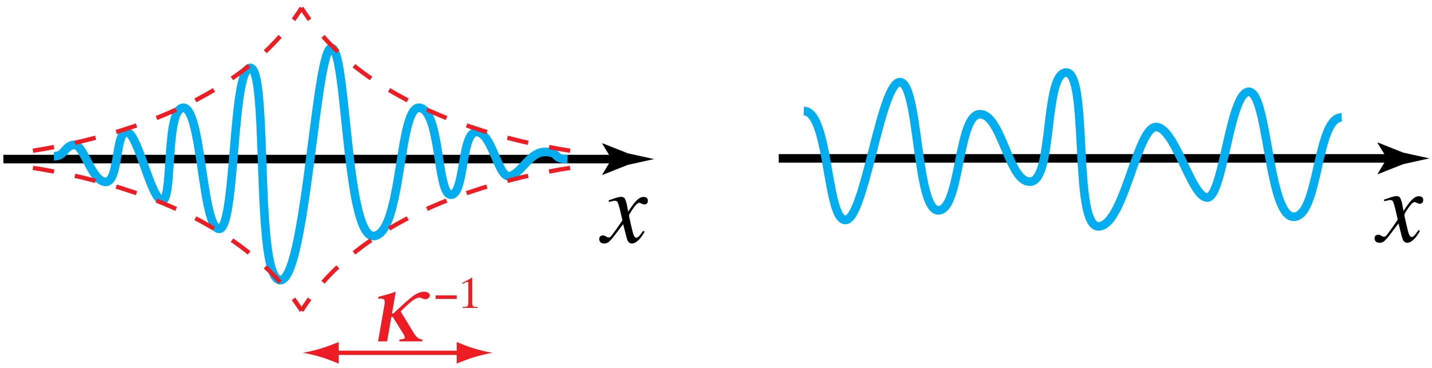
asymmetric hopping *vs.* randomness



Non-Hermitian delocalization

g=0: localized states

$\psi \sim e^{-\kappa|x|}$ $\kappa = 0$



(i) $g\uparrow$: delocalized
at $g = g_c$

(ii) delocalization ~
complex eigenvalue

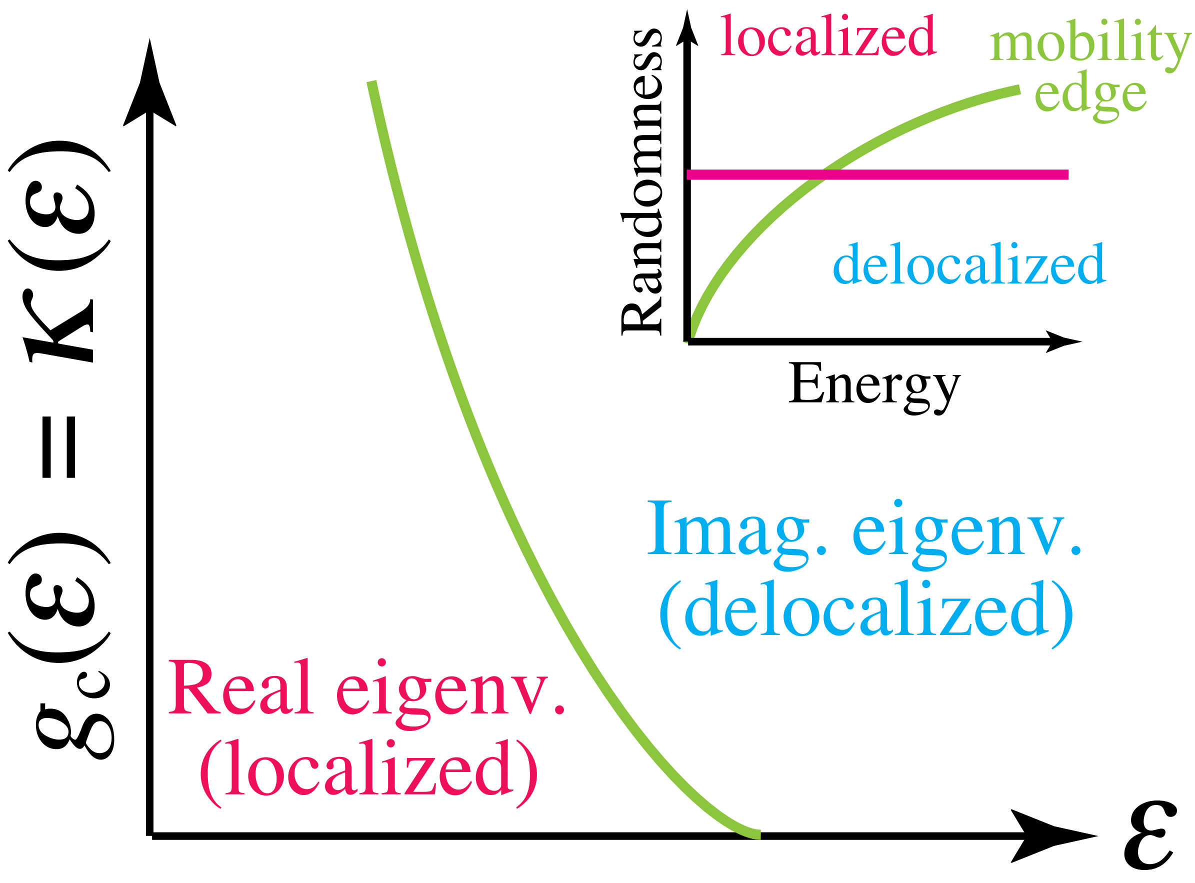
(iii) inverse localization
length: $\kappa = g_c$

Motivation

(1) Localization length of *Hermitian* Anderson model

Hermitian case ($g = 0$): $\psi_0 \sim e^{-\kappa|x|}$

Complex eigenvalue at $g = g_c$ ⟶ $\kappa = g_c$



How does it happen?

Imaginary gauge transformation

$$H_{\boldsymbol{g}} = \frac{(\boldsymbol{p} + i\boldsymbol{g})^2}{2m} + V(\boldsymbol{x})$$

$$e\boldsymbol{A} \leftrightarrow i\boldsymbol{g}$$

$$\boldsymbol{g} = \mathbf{0}: \quad H_0 \psi_0 = \varepsilon_0 \psi_0$$

⬇

$$\boldsymbol{g} \neq \mathbf{0}: \quad H_{\boldsymbol{g}} \psi_{\boldsymbol{g}} = \varepsilon \psi_{\boldsymbol{g}}$$

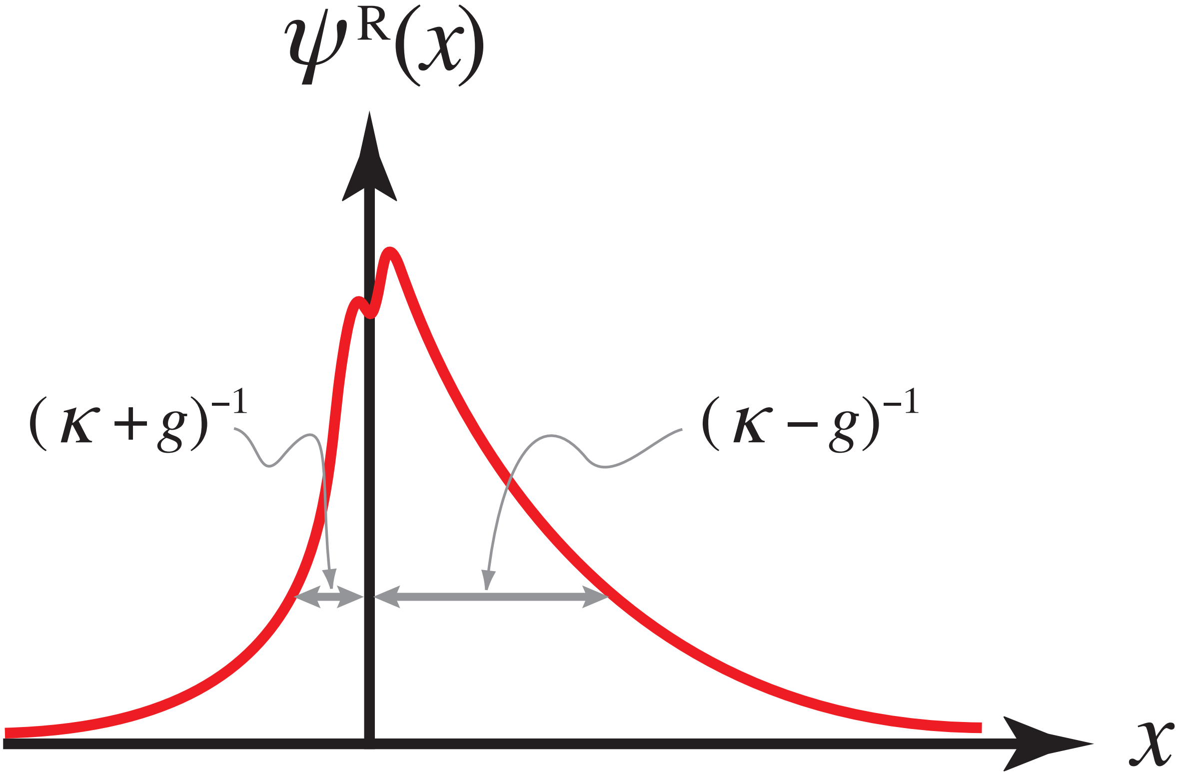
$$\psi_{\boldsymbol{g}} = \psi_0 e^{\boldsymbol{g} \cdot \boldsymbol{x}}$$

$$\varepsilon = \varepsilon_0 \quad : \text{real, fixed}$$

$$\psi_0 \sim e^{-\kappa|\boldsymbol{x}|}$$

$$\psi^{\mathrm{R}}_{\boldsymbol{g}} \sim e^{-\kappa|\boldsymbol{x}|+\boldsymbol{g}\cdot\boldsymbol{x}}$$

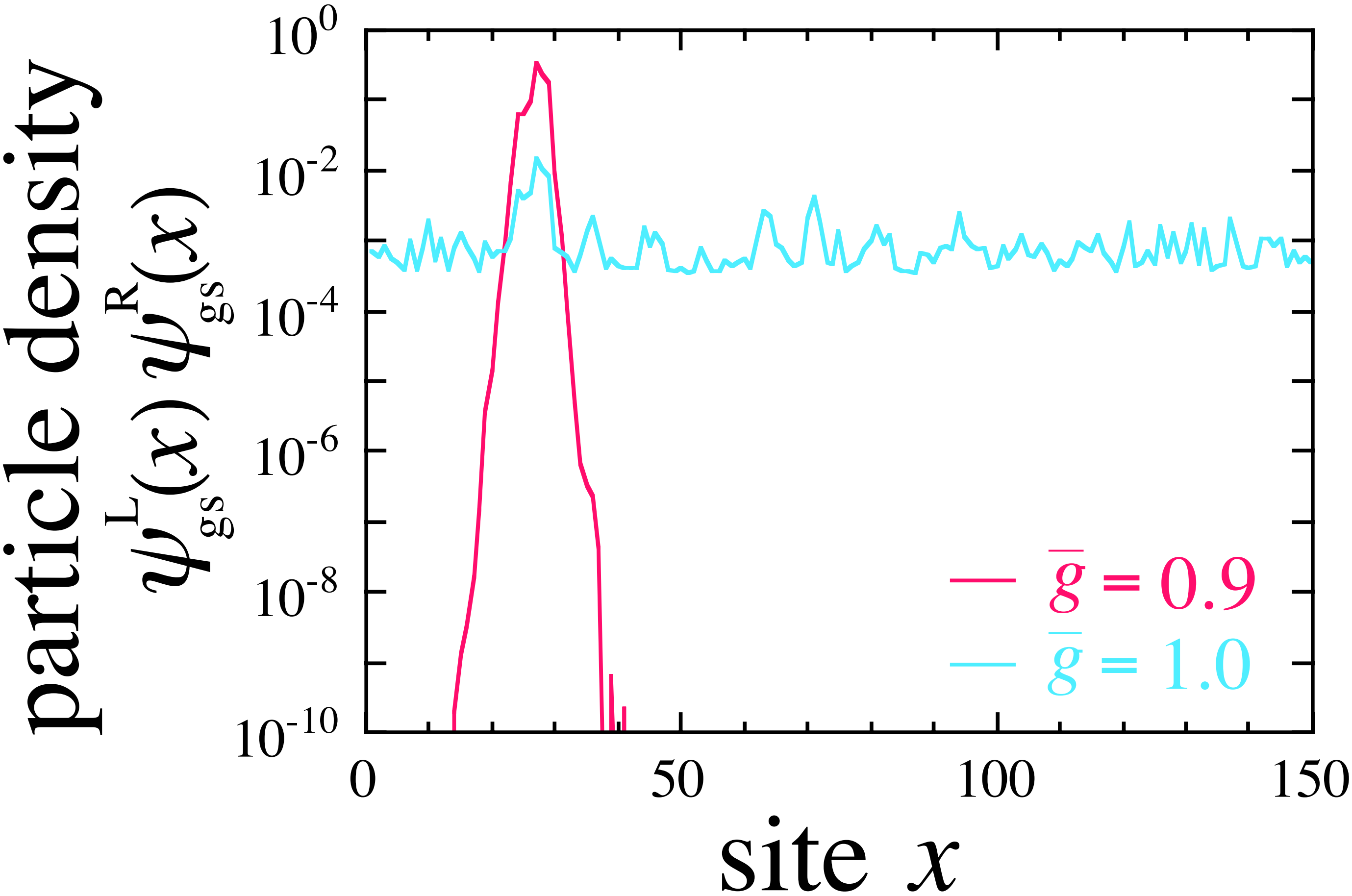
integrable for $|\boldsymbol{g}| < \kappa$ only



$$|\boldsymbol{g}| > \kappa: \ \psi^{\mathrm{R}} \sim \boldsymbol{e}^{i\boldsymbol{k}\cdot\boldsymbol{x}}, \ \varepsilon \sim \frac{(\boldsymbol{k}+i\boldsymbol{g})^2}{2m}$$

Delocalization of eigenfunction

1D: $L_x = 1000$ (Diagonalized 1000×1000 matrix)

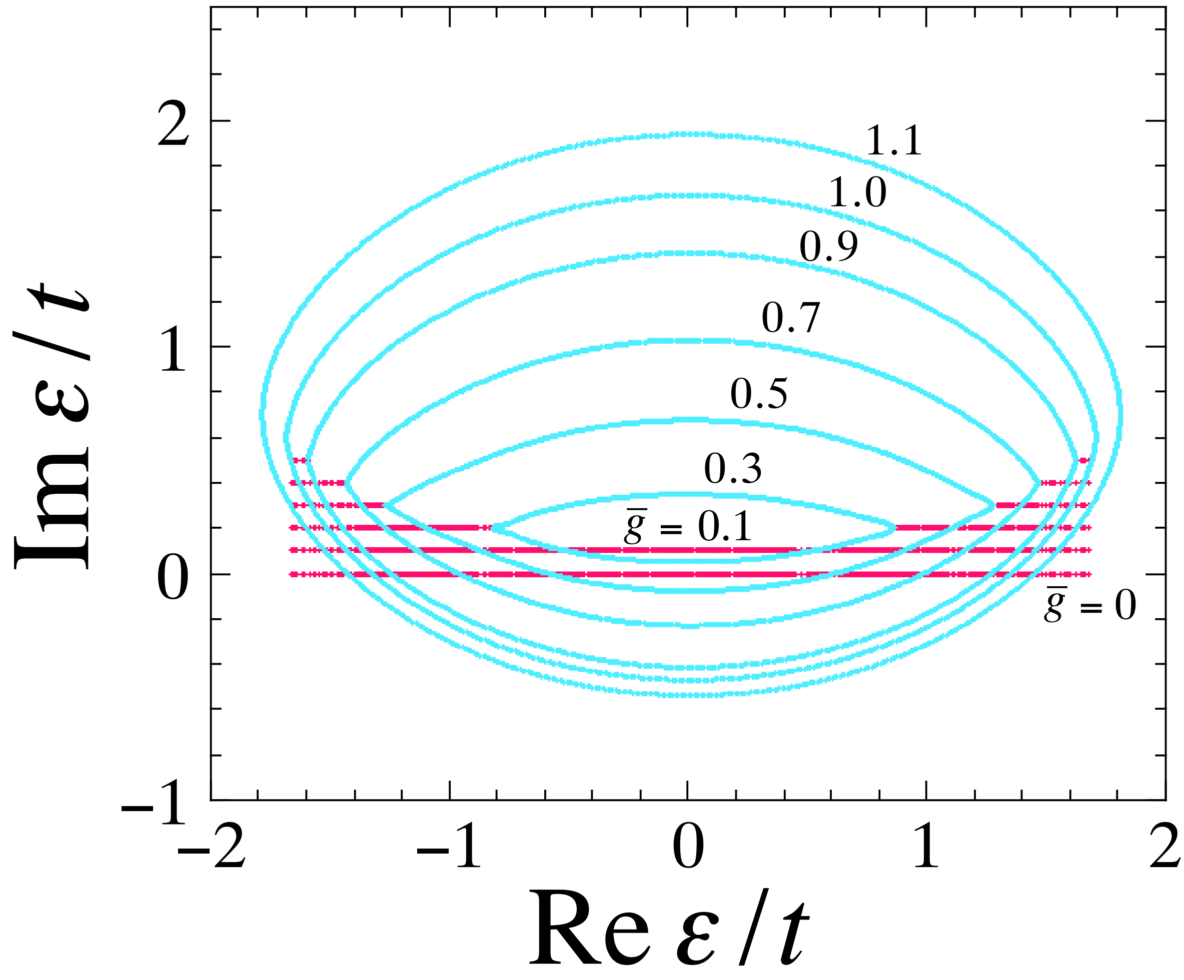


Note: Particle Density $\neq |\psi(x)|^2$

$$\left.\begin{array}{l} H\left|\psi^{R}\right\rangle = \varepsilon\left|\psi^{R}\right\rangle \\ \left\langle\psi^{L}\right|H = \varepsilon\left\langle\psi^{L}\right| \end{array}\right\} \rightarrow \left\langle\psi^{L}\right| \neq \left|\psi^{R}\right\rangle^{\dagger}$$

Complex spectrum and delocalization

1D: L_x = 1000 (Diagonalized 1000 × 1000 matrix)

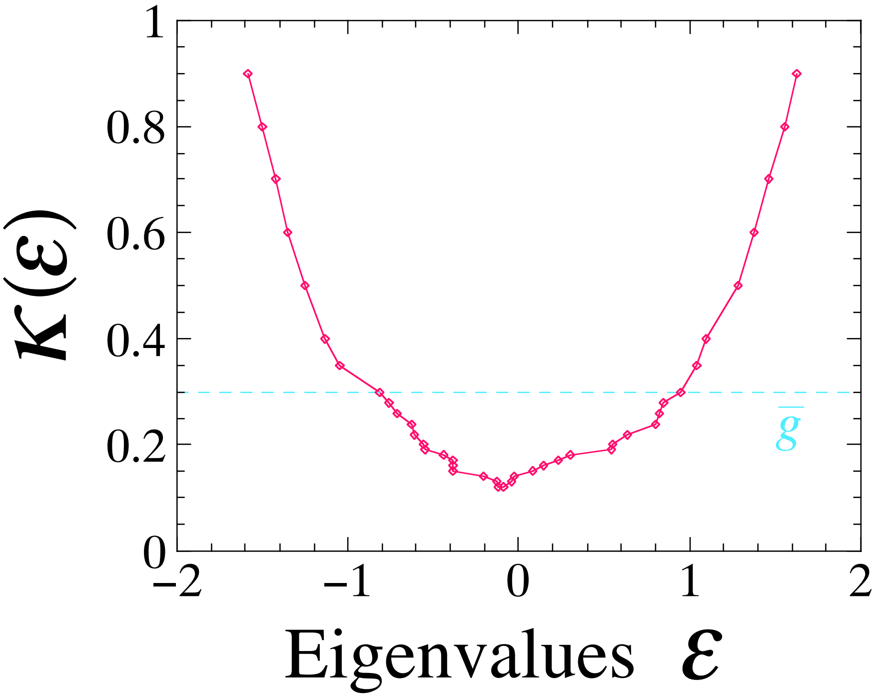


real eigenvalue
localized state
$\kappa > g$

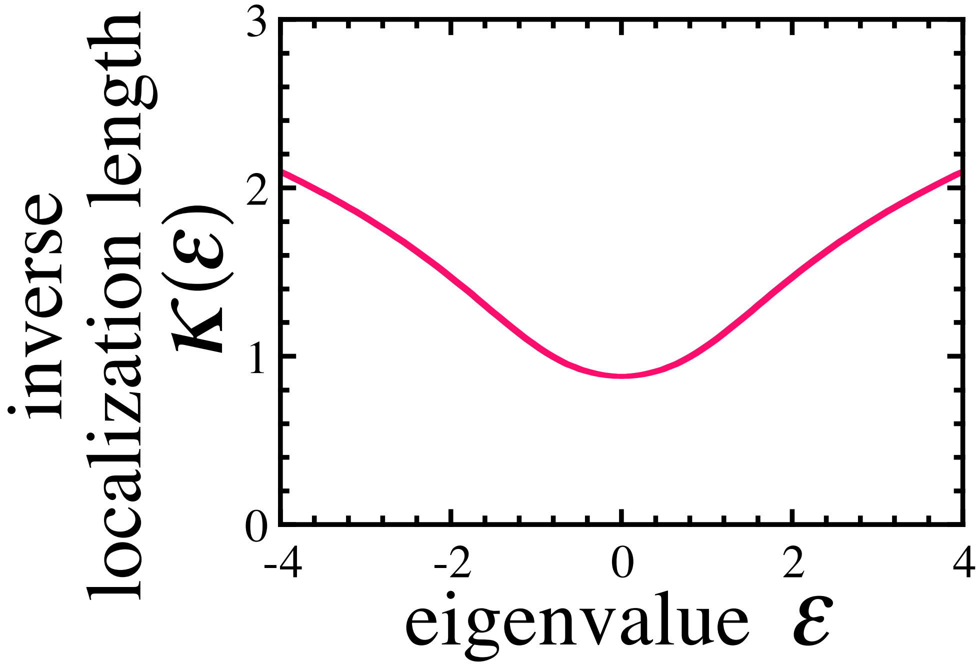
complex eigenv.
delocalized state
$\kappa < g$

Localization length

1D: $L_x = 1000$ (Diagonalized 1000×1000 matrix)

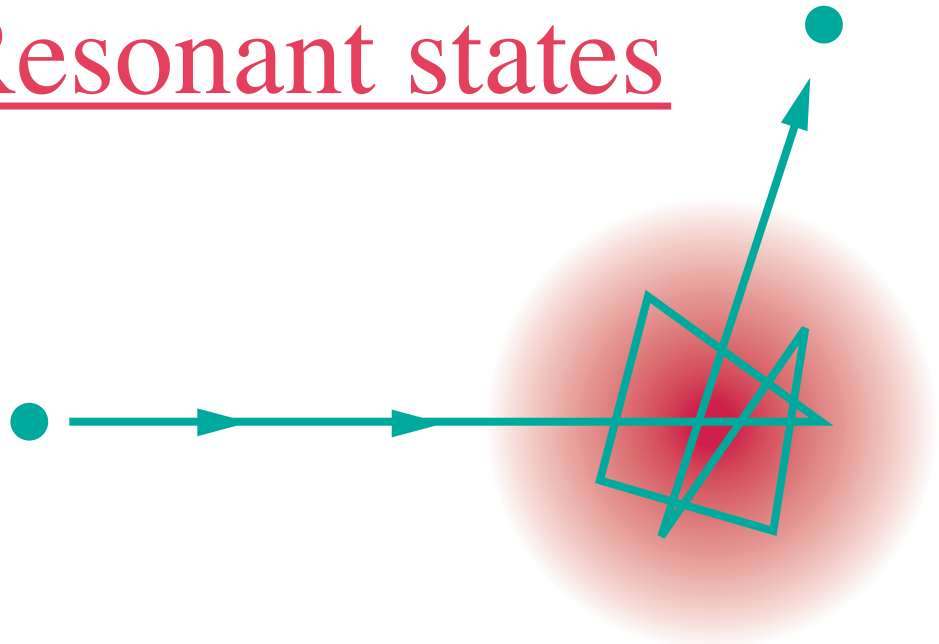


Lloyd model (Lorentzian random distribution):
Hirota & Ishii (1971), Brezin & Zee (1998)



Motivation

(2) Resonant states



$$E_{\mathrm{res}} = E_{\mathrm{r}} - i\Gamma/2 \approx \frac{(k - i\gamma)^2}{2m}$$

Γ^{-1}: Resonance lifetime

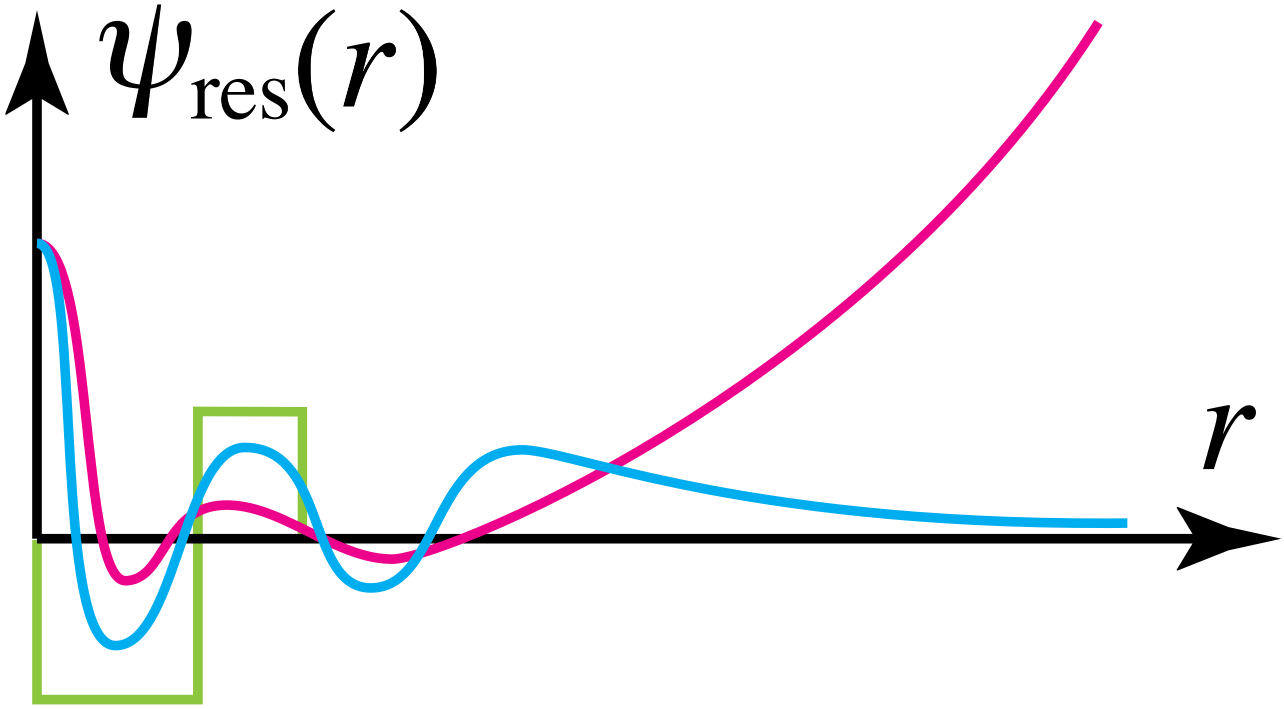
$$\psi_{\mathrm{res}}(r) \sim e^{i(k-i\gamma)r} \quad \text{as } r \to \infty$$

Unintegrable eigenfn.

$$H = \frac{(\boldsymbol{p}+i\boldsymbol{g})^2}{2m} + V(r); \quad \boldsymbol{g} \equiv -g_0 \frac{\boldsymbol{x}}{r}$$

$$\Downarrow$$

$$\psi_{\mathrm{res}}(r;g) \sim e^{ikr-(g_0-\gamma)r}$$



Aguilar, Balslev & Combes,
Comm. Math. Phys. **22** (1971)

$$\text{complex rotation: } \begin{cases} r \longrightarrow re^{i\theta} \\ p \longrightarrow pe^{-i\theta} \end{cases}$$

Other applications

- CDW pinning

Chen *et al*.,PRB **54**, 12798 (1996)

- Fokker-Planck equation

Nelson and Shnerb, PRE **58**, 1383 (1998)

- Dirac Fermion in random gauge field

Mudry *et al*. PRL **80**, 4257 (1998)

- *PT* symmetry

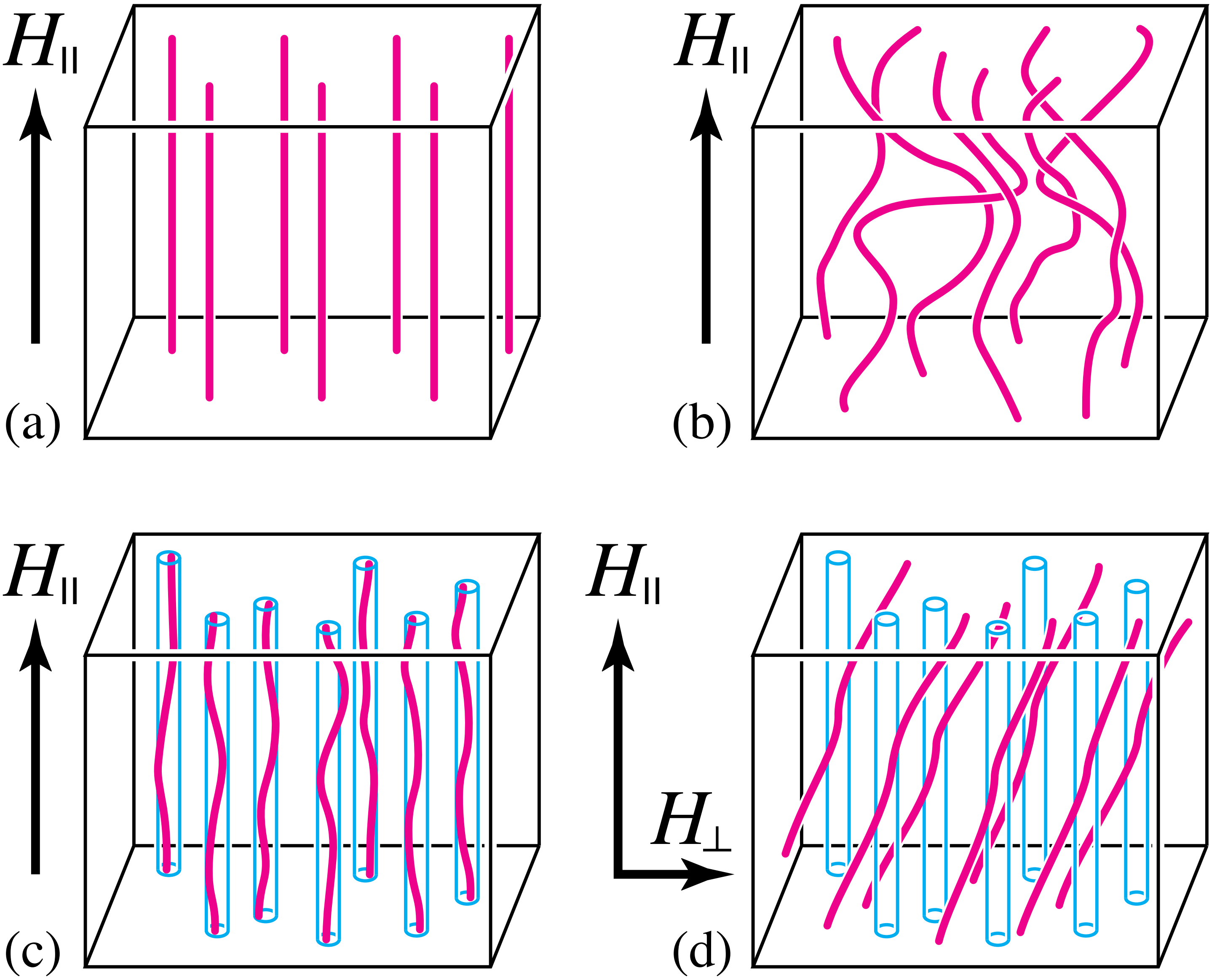
Bender and Boettcher, PRL **80**, 5243 (1998)
complex spectrum ↔ *PT* symmetry breaking

- Interactions

Lehrer and Nelson, cond-mat/9806016
Boson crystal and delocalization

Motivation

(3) Vortex pinning in high-T_c superconductors

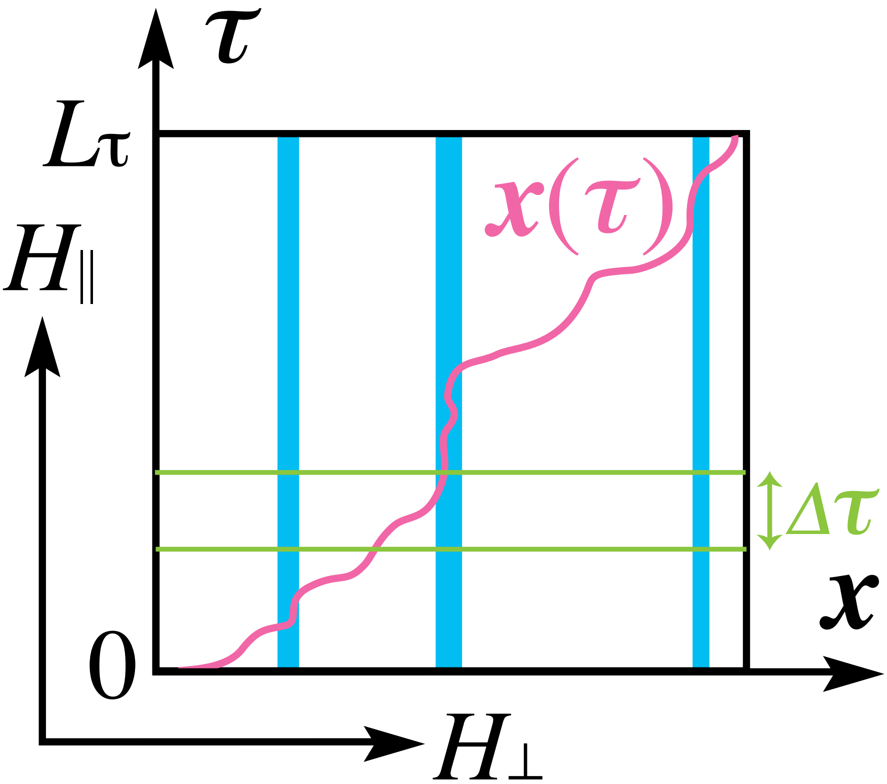


Path-integral mapping

$$E_{cl}[\boldsymbol{x}(\tau)] = \int_0^{L_\tau} d\tau \left[\frac{m}{2}\left(\frac{d\boldsymbol{x}}{d\tau}\right)^2 - \boldsymbol{g}\cdot\frac{d\boldsymbol{x}}{d\tau} + V(\boldsymbol{x}) \right]$$

$$\boldsymbol{g} \equiv \Phi_0 \boldsymbol{H}_\perp / (4\pi)$$

Nelson and Vinokur, PRB **48**, 13060 (1993)



$$\text{TM}: \; T(\Delta\tau) \approx \exp(-\Delta\tau\, H)$$

$$Z = \int \mathcal{D}\boldsymbol{x}\, e^{-E_{cl}[\boldsymbol{x}(\tau)]} = \langle \psi^f | e^{-L_\tau H} | \psi^i \rangle$$

Pinning and localization