

# Quantization of $\beta$ -Fermi-Pasta-Ulam Lattice with Nearest and Next-nearest Neighbour Interactions

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We quantize the  $\beta$ -Fermi-Pasta-Ulam (FPU) model with nearest and next-nearest neighbor interactions using a number conserving approximation and a numerically exact diagonalization method. Our numerical mean field bi-phonon spectrum shows excellent agreement with the analytic mean field results of Ivić and Tsironis ( Z. Ivić and G. Tsironis. *Physica D*, 216, 200, 2006), except for the wave vector at the midpoint of the Brillouin zone. We then relax the mean field approximation and calculate the eigenvalue spectrum of the full Hamiltonian. We show the existence of multi-phonon bound states and analyze the properties of these states by varying the system parameters. From the calculation of the spatial correlation function we then show that these multi-phonon bound states are particle like states with finite spatial correlation. Accordingly we identify these multi-phonon bound states as the quantum equivalent of the breather solutions of the corresponding classical FPU model. The four-phonon spectrum of the system is then obtained and its properties are studied. We then generalize the study to an extended range interaction and consider the quantization of the  $\beta$ -FPU model with next-nearest-neighbor interactions. We analyze the effect of the next-nearest-neighbor interactions on the eigenvalue spectrum and the correlation functions of the system.

Reference: A. Kibey, R. L. Sonone, B. Dey, and J. C. Eilbeck, Fermi-Pasta-Ulam lattice with nearest and next-nearest neighbour interactions, *Physica D* **294**, 43 (2015).