Exact spectral densities of non-Hermitian noise-plus-structure random matrices

Jacek Grela[1], Thomas Guhr[2]
Jagiellonian University, Krakow[1], *University of Duisburg-Essen, Duisburg*[2]
jacekgrela@gmail.com[1], thomas.guhr@uni-due.de[2]

We use supersymmetry to calculate exact spectral densities for a class of non-Hermitian complex random matrix models having the form $M = S + LXR$, where X is a random noise part X and S, L, R are fixed structure parts. This is a certain version of the "external field" random matrix models. We found two-fold integral formulas for arbitrary structural matrices. We investigate some special cases in detail and carry out numerical simulations. The presence or absence of a normality condition on S leads to a qualitatively different behaviour of the eigenvalue densities.

Relevant paper: http://arxiv.org/abs/1605.01159.

Dynamics of interaction of Peregrine solitons in the continuous nonlocal Schrödinger system with parity-time symmetry

Samit Kumar Gupta[1], Amarendra K. Sarma[2]
Indian Institute of Technology Guwahati[1], *Indian Institute of Technology Guwahati*[2]
samit.kumar.gupta@gmail.com[1]

Recently Ablowitz and Musslimani [*Phys. Rev. Lett.* ***110****, 064105 (2013)*] have proposed an integrable nonlocal nonlinear Schrödinger equation model where the standard third order nonlinearity $|\psi(x,z)|^2\psi(x,z)$ is replaced by its Parity-time-symmetric form: $\psi(x,z)\psi^*(-x,z)\psi(x,z)$. This equation admits Peregrine soliton solutions. This model also supports Peregrine rogue wave in the broken PT-phase for a single initial Peregrine soliton excitation. In this work, we report a numerical investigation of the interaction of two first order Peregrine solitons in both in-phase and out-of-phase conditions. We find that for the in-phase condition, in the unbroken PT-phase, the model yields the KM soliton breather when the transverse shift is small. On the other hand, in the broekn PT-phase it yields instability behaviors. In the out-of-phase condition, the model results in oblique propagation dynamics of the optical fields.

Tunneling in nearly integrable systems with a non-hermitian perturbation

Yasutaka Hanada[1], Akira Shudo[2] and Kensuke S. Ikeda[3]
Tokyo Metropolitan University[1,2], Ritsumeikan University[3]
hanada@tmu.ac.jp[1], shudo@tmu.ac.jp[2], ahoo@ike-dyn.ritsumei.ac.jp[3]

We study the tunneling effect in nearly integrable systems with a non-hermitian perturbation. In the integrable system such as a particle moving in the one-dimensional double well potential, the energy splitting ΔE caused by quantum tunneling is evaluated as

$$\Delta E \underset{\hbar \to 0}{\sim} e^{-S/\hbar}, \tag{1}$$

by the semiclassical (WKB) approximation, where the action S is determined by the classical quantity [1, 2]. On the other hand, in the systems under the periodic perturbation, the corresponding classical system becomes non-integrable. If one plots the energy splitting as a function of $1/\hbar$, it exhibits persistent enhancement from the prediction (1) accompanying spikes.

The spike can be interpreted as energetic resonance with excited states by photon absorption in the language of quantum dynamics, but it may be reinterpreted by the language of classical dynamics. The theory of resonance-assisted tunneling (RAT) have discussed the relation between the appearance of the spikes and the classical non-linear resonances, and then it claimed that the classical non-linear resonances create a bunch of spikes, which brings the enhancement of tunneling probability [3, 4].

The appearance of the spikes in the energy splitting has been considered as the origin of the enhancement of tunneling probability, but to make clear this issue, we introduce a weak non-hermitian perturbation which pushes the resonant states to the complex domain. By applying this perturbation, we found the spikes and the persistent enhancement have the different origin, and it was unveiled that the staircase-like structure is hidden in the energy splitting curve as a function of $1/\hbar$ [5].

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Quantum centrality testing on directed graphs via PT-symmetric quantum walks

Josh Izaac[1], Jingbo Wang[2], Xiaosong Ma[3]
School of Physics, The University of Western Australia, Crawley WA 6009[1,2]
School of Physics, Nanjing University, Nanjing, 210093, China[3]
josh.izaac@uwa.edu.au[1], jingbo.wang@uwa.edu.au[2], xiaosong.ma@nju.edu.cn[3]

Quantum walks are an important tool in the field of quantum information theory. Indeed as a method of universal quantum computation, they have motivated the creation of quantum algorithms that are faster and more efficient than their classical analogues, and provided a vital link between quantum computation and modeling complex quantum dynamical systems (for example, photosynthesis). However, one disadvantage of the quantum walk is the imposition of unitarity, due to the quantum nature of the walkers. As such, the conventional quantum walk is unable to model or analyze directed network structures, without either a) resulting in non-unitary dynamics, or b) modifying the framework. This serves as a particular hindrance in extending established quantum algorithms (e.g. quantum search, centrality measures, graph isomorphism) and quantum dynamical models to systems with direction/biased potentials (such as transport of electrons or excitons).

One such solution to this problem lies in the field of PT-symmetry, which offers the capability to perform quantum walks on directed graphs with non-Hermitian Hamiltonians whilst preserving the norm. In this presentation, we formalize a rigorous framework for continuous-time quantum walkers (CTQWs) on pseudo-Hermitian directed graph structures. This is then extended to the cases of multi-particle quantum walks and interdependent networks, before being utilized to measure vertex centrality in various small graphs – resulting in strong agreement with the classical PageRank algorithm, and in some cases even distinguishing vertex equivalence classes not identified by PageRank. Unlike previous quantum centrality measures, for a graph of N vertices this algorithm requires a Hilbert space of dimension N (compare this to the Szegedy quantum walk PageRank algorithm, which requires a N^2 dimensional Hilbert space), and without the classical decoherence required for open quantum systems. Furthermore, we show that this formalism is equivalent to considering an undirected, yet weighted, complete graph with self-loops, providing a structural interpretation that may lead to simple experimental implementation.

Scattering amplitudes for rationally extended PT symmetric complex potentials

Nisha Kumari[1], Rajesh Kumar Yadav[2], Avinash Khare[3], Bijan Bagchi[4], Bhabani Prasad Mandal[5]
Department of Physics, Banaras Hindu University, Varanasi-221005, India[1,5]*, Department of Physics, S. P. College, S. K. M. University, Dumka-814101, India*[2]*, Department of Physics, Savitribai Phule Pune University, Pune-411007, India*[3]*, Department of Physics, School of Natural Science, Shiv Nadar University, Greater Noida, UP-201314 India* [4]
nishaism0086@gmail.com [1], rajeshastrophysics@gmail.com[2], khare@physics.unipune.ac.in[3],bbagchi123@gmail.com[4], bhabani.mandal@gmail.com[5]

Scattering amplitudes for the rationally extended PT symmetric complex potentials In this paper, we consider the rational extensions of two different PT symmetric complex potentials namely the asymptotically vanishing Scarf II and asymptotically non-vanishing Rosen-Morse II [RM-II] potentials and obtain bound state eigenfunctions in terms of newly found exceptional X_m Jacobi polynomials and also some new type of orthogonal polynomials respectively. By considering the asymptotic behaviour of the exceptional polynomials, we obtain the reflection and transmission amplitudes for them and discuss the various novel properties of the corresponding amplitudes.

Time-evolution of quantum entropy associated with non-hermitian quantum dynamics

Hiroki Majima
Physics Department, General Education Division, Salesian Polytechnic
majima@salesio-sp.ac.jp

Non-hermitian Hamiltonians play an important role in the study of dissipative quantum systems. Traditionally non-hermitian Hamiltonians have been used to phenomenologically describe processes which lead to a non conservation on particle number, eg radioactive decay, scattering phenomena, and open quantum systems. In all such cases, the probability does not have to be conserved in general. Phase volume corresponds to entropy in classical theory, because the probability is represented by phase volume. Therefore, non-conservation of the probability causes changes of entropy

In this presentation, we consider the quantum entropy of systems described by non-hermitian Hamiltonians. The determinant of the density matrix in fact decays over time in non-hermitian Hamiltonian systems. From this, it can be presumed that the determinant of the density matrix in quantum systems corresponds to the phase volume in classical systems, from which the volume of a parallelepiped spanned by three vectors is the absolute value of the scalar triple product. According to this result, the determinant of the density matrix in quantum systems is comparable to phase volume compression in classical dissipative systems. We derive a relation between trace and determinant for the density matrix. The relation indicates that there is a difference relating to the degree of mixture between them. The phase volume is equivalent to entropy in classical theory. It implies that the determinant of the density matrix has relevance to entropy in quantum theory. We naturally show that quantum entropy is connected to the determinant of the density matrix in non-hermitian quantum systems which corresponds to phase volume compression in classical systems.

Next, we study an evolution equation with respect to a normalized density matrix in non-hermitian quantum systems. This evolution equation is derived by using a non-normalized density matrix. The generator of this evolution equation can be divided into two parts: The first part describes local unitary dynamics and the second term incoherent processes. In our previous work, we obtained the identity connect a change of the quantum entropy to a generator of the evolution equation. Applying the identity to the above evolution equation, we obtain a change in the quantum entropy attributed mainly to the incoherent part of the evolution equation for the density matrix. As a result, we can evaluate how the change in the quantum entropy is influenced from the incoherent part of the evolution equation in non-hermitian quantum systems.

$\mathcal{PT}$ symmetry for nonunitrary quantum walks and the extention to disordered systems

Ken Mochizuki, Hideaki Obuse
Department of Applied Physics, Hokkaido University
ken_mochizuki@eng.hokudai.ac.jp, hideaki.obuse@eng.hokudai.ac.jp

Recently, non-Hermitian systems with $\mathcal{PT}$ (combined parity and time-reversal) symmetry have been realized in various experiments, and many interesting phenomena have been observed in these systems. However, difficulty in controlling gain and loss effects makes most experimental systems have only a small number of elements. In contrast, quantum walks can overcome this difficulty.

Quantum walks recently attract attentions as a versatile platform for quantum computations and quantum simulations. Quantum walks describe quantum dynamics of particles (walkers) with several internal states by a time-evolution operator composed of coin operators, shift operators, and so on. The coin operator varies the internal states of walkers, and the shift operator changes the positions of walkers depending on the internal states. Quantum dynamics is described by acting the time-evolution operator to a quantum state at each time step. In 2012, a quantum walk built by optical fiber loops was experimentally implemented, in which highly tunable gain and loss effects are included with additional optical amplifiers [1]. Effects of gain and loss make the time-evolution operator nonunitary, which means the effective Hamiltonian is non-Hermitian. It is known that, in certain parameter regions, the system has entirely real quasienergy.

In the present work, we explicitly show that the reality of quasienergy obtained in Ref [1] stems from $\mathcal{PT}$ symmetry [2]. We find that parameters of operators must have correlations not only in spatial direction but also in time direction in order to preserve $\mathcal{PT}$ symmetry of the time-evolution operator. In addition, we numerically show that although parameters of the coin operator are random over the position space and the time-evolution operator does not hold $\mathcal{PT}$ symmetry in the explicit sense, the system can have entirely real quasienergy. Then, we discuss the reason why this occurs and study the effects of randomness on nonunitary quantum walks.

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A new type of PT-symmetric random matrix ensembles and PT-symmetric Quantum Chaos

Eva-Maria Graefe[1], **Steve Mudute-Ndumbe**[1], Sjoerd de Vries[1]
Department of Mathematics, Imperial College London[1]
e.m.graefe@imperial.ac.uk[1], steve.ndumbe10@imperial.ac.uk[1],
sjoerd.de-vries15@imperial.ac.uk[1]

We construct new ensembles of random matrices on the spaces of split-complex and split-quaternionic matrices, built using similar assumptions to the Gaussian Orthogonal, Unitary and Symplectic Ensembles, and compare some of the similarities and differences. We demonstrate that split-complex and split-quaternionic Hermitian matrices are isomorphic to PT-symmetric matrix Hamiltonians, and thus conjecture that these new ensembles can potentially be used as universality classes for PT-symmetric quantum systems whose closed equivalents are classically chaotic. We derive explicit results for the spectral densities of our new ensembles in the 2×2 case, and numerically identify properties in the more general $N \times N$ case. We further introduce PT-symmetric extensions of some paradigmatic models of quantum chaos and explore their spectral features.

$1/f$ noise in graphene thin films

Yuichi Ochiai[1], Nobuyuki Aoki[1], Katsuhiko Miyamoto[1], Takashige Omatsu[1], Tomohiro Yamaguchi[2], Kohji Ishibashi[2], Jonathan P. Bird[3], David K. Ferry[4]
Graduate School of Advanced Integration Science, Chiba University, Chiba, Japan[1], *Advanced Device Laboratory, Advanced Science Institute (RIKEN), Wako, Japan*[2], *Department of Electrical Engineering, University at Buffalo, New York, USA*[3], *Department of Electrical Engineering and CSSER, Arizona State University, Arizona, USA*[4]
ochiai@faculty.chiba-u.jp

There has been much interest in the transport properties in nano scale graphene thin films. In our work, monolayer and bilayer graphene devices have been fabricated as a very small antenna so as to apply for the terahertz (THz) detection. We focus specifically on the influence of THz irradiation as a noise property in graphene thin films. In our experiment at room temperature, two kinds of temperature coefficient have been observed in their resistance measurement as well as positive (metallic) or negative (semiconducting) temperature coefficient in room temperature, respectively, for monolayer or bilayer film. Then, we can use such dependence for our bolometric response as a clear detection of THz irradiation. The bolometric response has been characterized via changes in the spectral density of the $1/f$ resistance noise in the devices, as induced as a function of the incident THz power. Increasing radiation power, the spectral density has been found to show opposite dependence each other, metallic or semicondactiong, that are consistent with those exhibited by the opposite sign of the temperature coefficient of the devise resistance. Each bolometric response can be connected to a corresponding variation of the Hooge parameter as a function of the THz power. We would like to discuss on the $1/f$ resistance noise properties in graphene thin films.

Geometric fluctuation theorem

Kota Watanabe[1], Hisao Hayakawa[2]
Yukawa Institute for Theoretical Physics, Kyoto University
knabe@yukawa.kyoto-u.ac.jp[1], hisao@yukawa.kyoto-u.ac.jp[2]

Average bias is necessary for a steady current in a macroscopic system, but the average bias for a steady current is not always necessary in mesoscopic systems. Indeed, when a system is slowly and periodically modulated by several control parameters such as chemical potentials, gate voltages, and tunneling barriers, there exists a net average current without dc bias. This phenomenon is known as geometric pumping. The average current from the geometric pumping has been investigated by several researchers e.g. [1] in the adiabatic limit, but the forms of higher order cumulants and the fluctuation theorems are not known so far. The purpose of this study is (i) to clarify whether there is the fluctuation theorem in geometric pumping, and (ii) to identify the form of the fluctuation theorem as well as the fluctuation-dissipation relation in the lower order cumulants.

To accomplish our purpose, we use the Monte Carlo simulation to a spin-boson system with tunneling rate Γ. The result is presented in Fig. 1, where $P(q)$ is the distribution of charge transfer q with the control velocity Ω. Blue dots are data obtained from the simulation and the red line is the result of our theory. This result is similar to the steady fluctuation theorem under bias. In this presentation, we also demonstrate the existence of a generalized the fluctuation dissipation relation in this system.

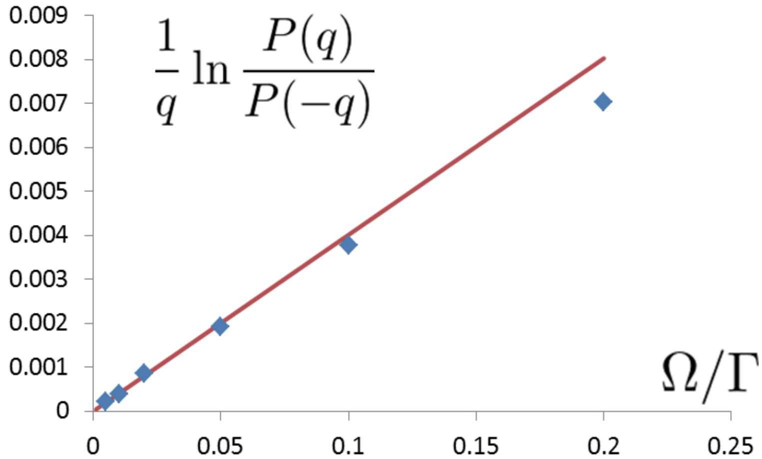

Figure 1:

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