Time-evolution of quantum entropy associated with non-hermitian quantum dynamics

Hiroki Majima
Physics Department, General Education Division, Salesian Polytechnic
majima@salesio-sp.ac.jp

Non-hermitian Hamiltonians play an important role in the study of dissipative quantum systems. Traditionally non-hermitian Hamiltonians have been used to phenomenologically describe processes which lead to a non conservation on particle number, eg radioactive decay, scattering phenomena, and open quantum systems. In all such cases, the probability does not have to be conserved in general. Phase volume corresponds to entropy in classical theory, because the probability is represented by phase volume. Therefore, non-conservation of the probability causes changes of entropy

In this presentation, we consider the quantum entropy of systems described by non-hermitian Hamiltonians. The determinant of the density matrix in fact decays over time in non-hermitian Hamiltonian systems. From this, it can be presumed that the determinant of the density matrix in quantum systems corresponds to the phase volume in classical systems, from which the volume of a parallelepiped spanned by three vectors is the absolute value of the scalar triple product. According to this result, the determinant of the density matrix in quantum systems is comparable to phase volume compression in classical dissipative systems. We derive a relation between trace and determinant for the density matrix. The relation indicates that there is a difference relating to the degree of mixture between them. The phase volume is equivalent to entropy in classical theory. It implies that the determinant of the density matrix has relevance to entropy in quantum theory. We naturally show that quantum entropy is connected to the determinant of the density matrix in non-hermitian quantum systems which corresponds to phase volume compression in classical systems.

Next, we study an evolution equation with respect to a normalized density matrix in non-hermitian quantum systems. This evolution equation is derived by using a non-normalized density matrix. The generator of this evolution equation can be divided into two parts: The first part describes local unitary dynamics and the second term incoherent processes. In our previous work, we obtained the identity connect a change of the quantum entropy to a generator of the evolution equation. Applying the identity to the above evolution equation, we obtain a change in the quantum entropy attributed mainly to the incoherent part of the evolution equation for the density matrix. As a result, we can evaluate how the change in the quantum entropy is influenced from the incoherent part of the evolution equation in non-hermitian quantum systems.