Quasi-parity and the unavoided crossing of energy levels in $\mathcal{PT}$-symmetric potentials

Géza Lévai[1]
Institute for Nuclear Research, Hungarian Academy of Sciences (MTA Atomki), Debrecen, Hungary[1]
`levai@atomki.mta.hu`[1]

The bound-state levels of one-dimensional quantum mechanical potentials are characterized by the n principal quantum number. In the case of real potentials the wave function can be chosen real, and n indicates the number of its nodes. The bound-state energies E_n strictly increase with increasing n, so the energy levels can never be degenerate. The n quantum number also appears for the $\mathcal{PT}$-symmetric version of these potentials. In contrast with the Hermitian case, there are some $\mathcal{PT}$-symmetric potentials that possess two series of normalizable states, which can be discriminated by the $q = \pm$ quasi-parity quantum number. It was also noticed that varying some potential parameters, energy levels with the *same* n and opposite q can merge and continue as complex conjugate energy eigenvalues, leading the the breakdown of $\mathcal{PT}$ symmetry. Furthermore, energy levels with *different* n and opposite q can cross, such that in the crossing point the corresponding wave functions become linearly dependent. The first examples for this mechanism were found for the $\mathcal{PT}$-symmetric harmonic oscillator [1] and Coulomb [2] potentials, while more recently this feature was also discussed for the $\mathcal{PT}$-symmetric Scarf II potential [3].

The question whether there are any further similar exactly solvable examples rasies naturally. Taking inspiration from this question, we discuss two further exactly solvable $\mathcal{PT}$-symmetric potentials, the energy spectrum of which are also known to be characterized by both the n and q quantum numbers. These are the $\mathcal{PT}$-symmetric versions of the generalized Ginocchio potential [4] and that of a four-parameter potential [5] that contains both the Scarf II and Rosen–Morse I potentials as special limits. An important difference between the two cases is that while the energy eigenvalues of the former one are written in closed form, and the q quantum number appears explicitly in the formulas, the energy eigenvalues of the latter potential are determined by the roots of a quartic algebraic equation, and q is involved in the procedure in an implicit form. We show that the unavoided crossing of energy levels occur in both cases. Furthermore, we demonstrate that this feature is also valid for the generic Natanzon-class potentials [6]. Some further consequences of this finding are also outlined.

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