Quantum centrality testing on directed graphs via PT-symmetric quantum walks

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Quantum walks are an important tool in the field of quantum information theory. Indeed as a method of universal quantum computation, they have motivated the creation of quantum algorithms that are faster and more efficient than their classical analogues, and provided a vital link between quantum computation and modeling complex quantum dynamical systems (for example, photosynthesis). However, one disadvantage of the quantum walk is the imposition of unitarity, due to the quantum nature of the walkers. As such, the conventional quantum walk is unable to model or analyze directed network structures, without either a) resulting in non-unitary dynamics, or b) modifying the framework. This serves as a particular hindrance in extending established quantum algorithms (e.g. quantum search, centrality measures, graph isomorphism) and quantum dynamical models to systems with direction/biased potentials (such as transport of electrons or excitons).

One such solution to this problem lies in the field of PT-symmetry, which offers the capability to perform quantum walks on directed graphs with non-Hermitian Hamiltonians whilst preserving the norm. In this presentation, we formalize a rigorous framework for continuous-time quantum walkers (CTQWs) on pseudo-Hermitian directed graph structures. This is then extended to the cases of multi-particle quantum walks and interdependent networks, before being utilized to measure vertex centrality in various small graphs – resulting in strong agreement with the classical PageRank algorithm, and in some cases even distinguishing vertex equivalence classes not identified by PageRank. Unlike previous quantum centrality measures, for a graph of N vertices this algorithm requires a Hilbert space of dimension N (compare this to the Szegedy quantum walk PageRank algorithm, which requires a N^2 dimensional Hilbert space), and without the classical decoherence required for open quantum systems. Furthermore, we show that this formalism is equivalent to considering an undirected, yet weighted, complete graph with self-loops, providing a structural interpretation that may lead to simple experimental implementation.