Non-Hermitian Localization and Delocalization in the generalized Feinberg-Zee Model

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We consider the following non-Hermitian tight-binding model [1]:

$$H = \sum_{x=1}^{L} \left(s_x |x+1\rangle\langle x| + t_x |x\rangle\langle x+1| \right), \tag{1}$$

where we choose the hopping elements to the right, $\{s_x\}$, and to the left, $\{t_x\}$, independently randomly from ± 1. We also impose the periodic boundary condition. Such an $L \times L$ matrix can represent, for example, a neural network in which the synapse connection from the xth neuron to the $x+1$th one and the connection in the opposite direction may be independently excitatory or inhibitory. The model has been introduced by Feinberg and Zee [2] in the context of random matrix theory and is known to show a fractal-like spectrum.

In order to know the localization nature of the eigenvectors of the Feinberg-Zee model, we introduce a non-Hermitian gauge field g in the spirit of the Hatano-Nelson model [3], as follows:

$$H = \sum_{x=1}^{L} \left(e^{g} s_x |x+1\rangle\langle x| + e^{-g} t_x |x\rangle\langle x+1| \right). \tag{2}$$

We found that a hole emerges in the middle of the spectrum and grows as we increase g [1]. We argue that the eigenstates on the rim of the hole for g has the inverse localization length equals to g. This is in consistent with a recent numerical calculation [4] based on the Chebyshev-polynomial expansion of the inverse localization length.

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References

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