Non-Hermitian Localization and Delocalization in the generalized Feinberg-Zee Model

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We consider the following non-Hermitian tight-binding model [1]:

\[ H = \sum_{x=1}^{L} (s_x|x + 1\rangle\langle x| + t_x|x\rangle\langle x + 1|), \]  \(1\)

where we choose the hopping elements to the right, \(\{s_x\}\), and to the left, \(\{t_x\}\), independently randomly from \(\pm 1\). We also impose the periodic boundary condition. Such an \(L\times L\) matrix can represent, for example, a neural network in which the synapse connection from the \(x\)th neuron to the \(x + 1\)th one and the connection in the opposite direction may be independently excitatory or inhibitory. The model has been introduced by Feinberg and Zee [2] in the context of random matrix theory and is known to show a fractal-like spectrum.

In order to know the localization nature of the eigenvectors of the Feinberg-Zee model, we introduce a non-Hermitian gauge field \(g\) in the spirit of the Hatano-Nelson model [3], as follows:

\[ H = \sum_{x=1}^{L} (e^g s_x|x + 1\rangle\langle x| + e^{-g} t_x|x\rangle\langle x + 1|). \]  \(2\)

We found that a hole emerges in the middle of the spectrum and grows as we increase \(g\) [1]. We argue that the eigenstates on the rim of the hole for \(g\) has the inverse localization length equals to \(g\). This is in consistent with a recent numerical calculation [4] based on the Chebyshev-polynomial expansion of the inverse localization length.

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References