

# $\mathcal{PT}$ -symmetry and integrability as reality conditions for complex solitons

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We report complex  $\mathcal{PT}$ -symmetric multi-soliton solutions to the Korteweg de-Vries, modified Korteweg de-Vries and complex sine-Gordon equation. The solutions are obtainable from Hirota's direct method or multiple Bäcklund transformations. We show that complex solitons possess the new feature of allowing to construct compound solutions that asymptotically contain one-soliton solutions, with each of them possessing the same amount of finite real energy. We demonstrate how these solutions originate from degenerate energy solutions of the Schrödinger equation, which is technically achieved by the application of Darboux-Crum transformations involving Jordan states with suitable regularizing shifts. We compute the time-delays resulting in a multi-soliton scattering and argue that  $\mathcal{PT}$ -symmetry together with integrability guarantees the reality of all conserved charges.