The $-x^4$ potential

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The upside-down potential $V(x) = -x^4$ has totally different spectra depending on how it is defined. As a conventional quantum-mechanical potential, $V(x)$ has a continuous spectrum. If $V(x)$ is defined as

$$V(X) \equiv \lim{}_{\theta:\ 0\to\pi} e^{i\theta} x^4,$$

its eigenvalues are complex. However, if $V(x)$ is defined as a $\mathcal{PT}$-symmetric potential

$$V(x) \equiv \lim{}_{\varepsilon:\ 0\to 2} x^2 (ix)^{\varepsilon},$$

its eigenvalues are *real, positive, and discrete*.

In this talk we show heuristically and prove rigorously that the $\mathcal{PT}$-symmetric version of $V(x) = -x^4$ has a positive discrete spectrum. We then propose a laboratory experiment to verify this result.