YITP Workshop

PHHQP16

Progress in Quantum Physics with Non-Hermitian Operators

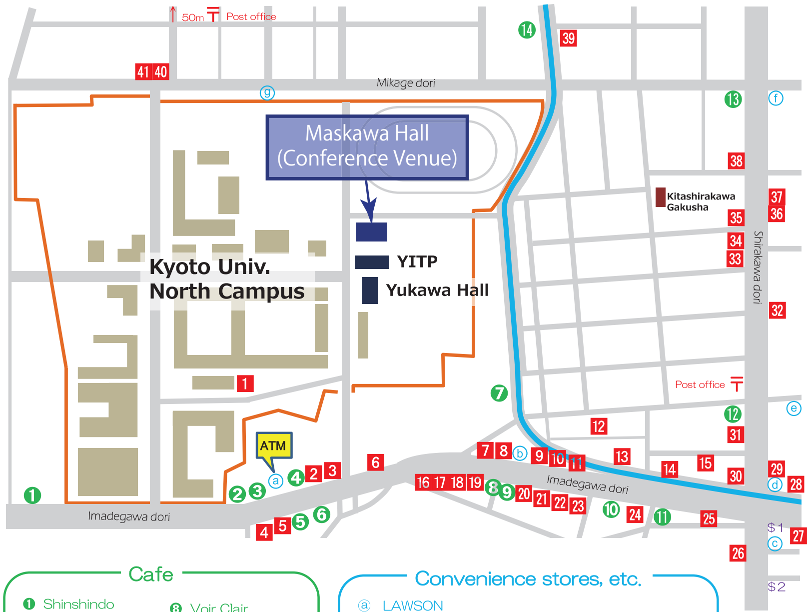
August 8–12, 2016

Yukawa Institute for Theoretical Physics, Kyoto University

Abstract book

May 2016 ver.1

Restaurants around YITP (east side)



Meals, etc.

1. Cafeteria
2. Yoshidaya (Ramen)
3. Matsunosuke (Japanese, Western)
4. Ryumon (Chinese)
5. Junsai (Izakaya)
6. Kenken (Set meal) Bintan (Asian)
7. Hakusui (Chinese)
8. Charlie's Yume-Hiko (Western)
9. Tenkaippin (Ramen)
10. Minori (Soba)
11. Taritappa、Barb (Bar)
12. Thiopepe (Spanish)
13. Eisuke (Original dishes)
14. Masutani (Ramen)
15. Isshin (Hot pot, Barbecue)
16. Matsuo (Saraudon • Chanpon)
17. Kurarishokubo (Western)
18. MAHA (Indian food)
19. GOYA (Asian food)
20. e-tokoya (Izakaya)
21. Kisuke (Yakitori)
22. Himawari (Ramen)
23. Sushijin (Sushi)
24. Primo piano (Italian)
25. Otayan (Curry-udon)
26. Shirakawa (Ramen)
27. Daiginshokudo (Set meal)
28. Kagitomi (Soba)
29. Tokurien (Chinese)
30. Sanyukyo (Japanese • Kaiseki)
31. Rihaku (Chinese)
32. Kanidoraku (Crab • Kaiseki)
33. Uokane (Tenpura)
34. Akatsuki (Ramen)
35. Tomoe (Sushi)
36. Kimasshi (Okonomiyaki)
37. Tonryu (Ramen)
38. KFC (Fried chicken)
39. ReaBon (French)
40. Washinro (Chinise)
41. Hiraganakan (Western)

Cafe

1. Shinshindo
2. TOSCA
3. Cafe Collection
4. Sako Boon
5. Ogawa Coffee
6. Gyokuran
7. Shizuku
8. Voir Clair
9. Arabica
10. Ben's Cafe
11. So-an cafe
12. World Coffee
13. Juenesse
14. Rihou

Convenience stores, etc.

- ⓐ LAWSON
- ⓑ LAWSON
- ⓒ Ginkakuji-candy (Sweets, Popsicles)
- ⓓ Un jour Momonoki (Bakery)
- ⓔ CONA-CONA (Bakery)
- ⓕ Circle K
- ⓖ FamilyMart

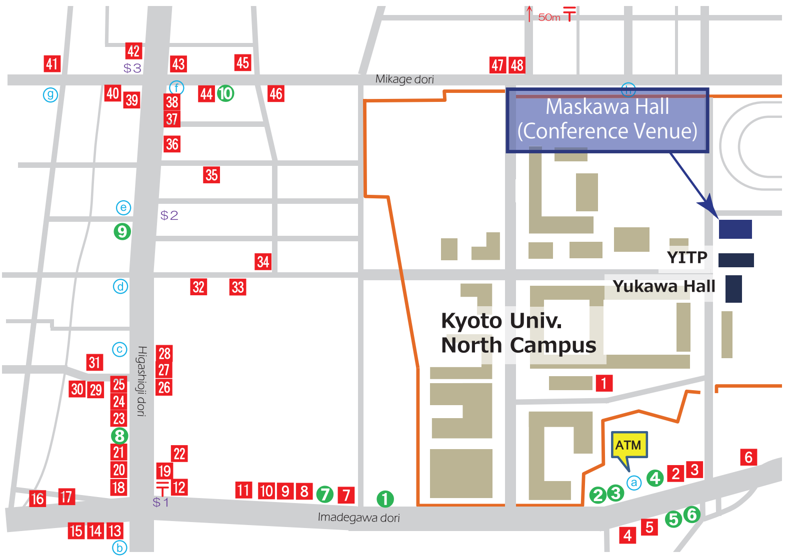
Currency exchange

$1 Bank of Kyoto
JPY ⇄ USD (only for a small amount)
10:30-14:30, passport, hotel address, telephone no.

$2 THE KYOTO SHINKIN BANK
JPY ⇄ USD (only for a small amount)
10:00 – 15;00, passport

May 2016 ver.1

Restaurants around YITP (west side)



Meals, etc.

1. Cafeteria
2. Yoshidaya (Ramen)
3. Matsunosuke (Japanese, Western)
4. Ryumon (Chinese)
5. Junsai (Izakaya)
6. Kenken (Set meal) Bintan (Asian)
7. Kohshin (Chinese)
8. Sujata (Vegetarian food)
9. Raju (Indian food、halal)
10. hi-lite (Set meal)
11. Salman (Asian food)
12. Kushihachi (Kushiage、Kushikatsu)
13. Subway (Sandwich)
14. Matsuya (Beef bowl、Soba、Udon)
15. CoCo ICHIBANYA (Curry)
16. Kureshima (Izakaya)
17. McDonald's (Hamburger)
18. Ohsho (Dumpling・Chinese)
19. Tsukemenman (Tsukemen)
20. Sukiya (Bowls、Curry)
21. Denmaru (Ramen)
22. Bono (Kushiage、Skewers)
23. Kiramekijapan (Ramen)
24. James Kitchen (Western)
25. Maru (Izakaya)
26. Momoya (Pork cutlet、cold noodle)
27. Tsumugi (Sushi)
28. Ma Cantine (Continental)
29. Sharaku (Izakaya)
30. Ekatei (Barbecue)
31. Mon (Izakaya • Hot pot)
32. Taruhachi (Izakaya)
33. Kicchan (Korean food)
34. Sumiyoshi (Izakaya)
35. Yampao (Original dishes, • Izakaya)
36. Rairaihanten (Ramen)
37. Narikoma (Sushi)
38. Choukouhen (Chinese)
39. Kitchen Satonoya (Barbecue)
40. Houenbimi (Chinese)
41. Kuishinbo (Okonomiyaki)
42. Menchumuchu (Ramen)
43. Kasho (Chinese)
44. Okuda (Pork cutlet)
45. Jidaiya (Izakaya • Yakitori)
46. La Part Dieu (French)
47. Hiraganakan (Western)
48. Washinro (Chinese)

Cafe

1. Shinshindo
2. TOSCA
3. Cafe Collection
4. Sako Boon
5. Ogawa Coffee
6. Gyokuran
7. ALPINE
8. HOME TOWN
9. Colorado
10. Kenya

Convenience stores, etc.

- ⓐ LAWSON
- ⓑ LAWSON
- ⓒ LAWSON STORE 100
- ⓓ FamilyMart
- ⓔ Holy Land (Bakery)
- ⓕ LAWSON
- ⓖ Takonaguri (Takoyaki)
- ⓗ FamilyMart

Currency exchange

$1 Japan Post Bank
JPY ⇄ USD, EUR (only for a small amount)
9:00-16:00, passport,

$2 Kyoto Chuo Shinkin Bank
JPY ⇄ USD (only for a small amount)
10:00-14:00, passport,

$3 Bank of Kyoto
JPY ⇄ USD (only for a small amount)
10:30-14:30, passport, hotel address, telephone no.

	August 8	August 9	August 10	August 11	August 12
9:00	registration				
9:20	opening				
9:30	Bender	Brody	Wang	Joglekar	Fring
10:20	break	break	break	break	break
10:40	Bagarello	Shudo	Günther	Mostafazadeh	Feinberg
11:30	Levai	Obuse	Tanaka	Lazauskas	Hatano
11:55	Yadav	Maamache	Mebarki	Luna-Acosta	Pato
12:20	lunch	lunch	lunch	lunch	lunch
13:30	Graefe	Heiss	excursion	poster session*	Shafarevich
14:20	Siegl	Petrosky			Bounames
14:45		Kanki			Sokolov
15:10	break	break		break	break
15:30	Michel	Garmon		Mandal	Zonjil
15:55	Oishi	Melgaard			
16:20	Bagchi	Sato		Sonone	conclusion
16:45		Ganguly		Ishkhanyan	
17:10					
			18:00 banquet		

*Poster presenters: Grela, Gupta, Hanada, Izaac, Kumar, Kumari, Majima, Mochizuki, Mudute-Ndumbe

Part I: Oral Presentations

The $-x^4$ potential

Carl M. Bender
Physics Department, Washington University in St. Louis
cmb@wustl.edu

The upside-down potential $V(x) = -x^4$ has totally different spectra depending on how it is defined. As a conventional quantum-mechanical potential, $V(x)$ has a continuous spectrum. If $V(x)$ is defined as

$$V(X) \equiv \lim{}_{\theta:\ 0\to\pi} e^{i\theta} x^4,$$

its eigenvalues are complex. However, if $V(x)$ is defined as a $\mathcal{PT}$-symmetric potential

$$V(x) \equiv \lim{}_{\varepsilon:\ 0\to 2} x^2 (ix)^{\varepsilon},$$

its eigenvalues are *real, positive, and discrete.*

In this talk we show heuristically and prove rigorously that the $\mathcal{PT}$-symmetric version of $V(x) = -x^4$ has a positive discrete spectrum. We then propose a laboratory experiment to verify this result.

Deformed graphene

Fabio Bagarello
Dipartimento di Energia, Ingegneria dell'Informazione e Modelli Matematici,
Scuola Politecnica Ingegneria, Università di Palermo,
I-90128 Palermo, Italy
`fabio.bagarello@unipa.it`

We propose two deformed versions of the graphene tight-binding model under a magnetic field: the first one is obtained by replacing bosonic with pseudo-bosonic operators, while the other is constructed by inserting a suitable deformation parameter V in the original two-by-two Hamiltonian. Both these versions involve non Hermitian Hamiltonians. We analyze the structure of the spectra and the eigenvectors of the Hamiltonians around the K and K' points. In particular, we show that, when $V \neq 0$, the completeness of the eigenvector sets is lost. We also discuss the biorthogonality of the eigenvectors.

Quasi-parity and the unavoided crossing of energy levels in $\mathcal{PT}$-symmetric potentials

Géza Lévai
Institute for Nuclear Research, Hungarian Academy of Sciences (MTA Atomki), Debrecen, Hungary
levai@atomki.mta.hu

The bound-state levels of one-dimensional quantum mechanical potentials are characterized by the n principal quantum number. In the case of real potentials the wave function can be chosen real, and n indicates the number of its nodes. The bound-state energies E_n strictly increase with increasing n, so the energy levels can never be degenerate. The n quantum number also appears for the $\mathcal{PT}$-symmetric version of these potentials. In contrast with the Hermitian case, there are some $\mathcal{PT}$-symmetric potentials that possess two series of normalizable states, which can be discriminated by the $q = \pm$ quasi-parity quantum number. It was also noticed that varying some potential parameters, energy levels with the *same* n and opposite q can merge and continue as complex conjugate energy eigenvalues, leading the the breakdown of $\mathcal{PT}$ symmetry. Furthermore, energy levels with *different* n and opposite q can cross, such that in the crossing point the corresponding wave functions become linearly dependent. The first examples for this mechanism were found for the $\mathcal{PT}$-symmetric harmonic oscillator [1] and Coulomb [2] potentials, while more recently this feature was also discussed for the $\mathcal{PT}$-symmetric Scarf II potential [3].

The question whether there are any further similar exactly solvable examples rasies naturally. Taking inspiration from this question, we discuss two further exactly solvable $\mathcal{PT}$-symmetric potentials, the energy spectrum of which are also known to be characterized by both the n and q quantum numbers. These are the $\mathcal{PT}$-symmetric versions of the generalized Ginocchio potential [4] and that of a four-parameter potential [5] that contains both the Scarf II and Rosen–Morse I potentials as special limits. An important difference between the two cases is that while the energy eigenvalues of the former one are written in closed form, and the q quantum number appears explicitly in the formulas, the energy eigenvalues of the latter potential are determined by the roots of a quartic algebraic equation, and q is involved in the procedure in an implicit form. We show that the unavoided crossing of energy levels occur in both cases. Furthermore, we demonstrate that this feature is also valid for the generic Natanzon-class potentials [6]. Some further consequences of this finding are also outlined.

[1] M. Znojil, Phys. Lett. A **259** (1999) 220.
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[4] G. Lévai, A. Sinha and P. Roy, J. Phys. A:Math. Gen. **36** (2003) 7611.
[5] G. Lévai, J. Phys. A:Math. Theor. **45** (2012) 444020.
[6] G. Lévai, Int. J. Theor. Phys. **54** (2015) 2724.

Dirichlet spectrum of the complex PT-symmetric Bender-Boettcher potential: $\mathbf{V(x) = -(ix)^{\nu}}$

Sachin Kumar
Theoretical Physics Section, Bhabha Atomic Research Centre, Trombay, Mumbai, 400085, India
sachinv@barc.gov.in

In this paper, we discuss the parametric symmetries in different exactly solvable systems characterized by real or complex PT symmetric potentials. We focus our attention on the conventional potentials such as the generalized Pöschl Teller (GPT), Scarf-I and PT symmetric Scarf-II which are invariant under certain parametric transformations. The resulting set of potentials are shown to yield a completely different behavior of the bound state solutions. Further the supersymmetric (SUSY) partner potentials acquire different forms under such parametric transformations leading to new sets of exactly solvable real and PT symmetric complex potentials. These potentials are also observed to be shape invariant (SI) in nature. We subsequently take up a study of the newly discovered rationally extended SI Potentials, corresponding to the above mentioned conventional potentials, whose bound state solutions are associated with the exceptional orthogonal polynomials (EOPs). We discuss the transformations of the corresponding Casimir operator employing the properties of the $so(2,1)$ algebra.

A new semiclassical propagator for non-Hermitian quantum systems

Eva-Maria Graefe[1], Alexander Rush
Department of Mathematics, Imperial College London
e.m.graefe@imperial.ac.uk[1]

Gaussian states form an over-complete basis of the Hilbert space of square integrable functions. This fact can be used to devise propagation schemes for arbitrary quantum states expanded into Gaussians. One very successful approach for Hermitian systems is Heller's *hybrid dynamics*. In this method the dynamics of the individual Gaussian components are approximated by the semiclassical dynamics valid for short times, and the propagated state is re-expanded into Gaussians at short time intervals. In recent years the time-evolution of Gaussian wave packets in the short-time limit has been extended to non-Hermitian systems. This offers the opportunity to devise analogous numerical propagation schemes for the dynamics of non-Hermitian quantum systems, which will be discussed in the present talk. We will give a brief overview over Heller's hybrid method in the Hermitian case, and the short time dynamics of Gaussian wave packets generated by non-Hermitian Hamiltonians. Then we will present examples of the new numerical method in non-Hermitian systems.

Approximations of spectra of Schrödinger operators with complex potentials on $\mathbb{R}^d$

Petr Siegl
Mathematical Institute, University of Bern, Switzerland
petr.siegl@math.unibe.ch

We study spectral approximations of Schrödinger operators $T = -\Delta + Q$ with complex potentials on $\Omega = \mathbb{R}^d$, or exterior domains $\Omega \subset \mathbb{R}^d$, by domain truncation. Our weak assumptions cover wide classes of potentials Q for which T has discrete spectrum, of approximating domains Ω_n, and of boundary conditions on $\partial\Omega_n$ such as mixed Dirichlet/Robin type. In particular, $\Re Q$ need not be bounded from below and Q may be singular. We prove generalized norm resolvent convergence and spectral exactness, i.e. approximation of *all* eigenvalues of T by those of the truncated operators T_n *without* spectral pollution. Moreover, we estimate the eigenvalue convergence rate and prove convergence of pseudospectra. Our results are illustrated by numerical computations for several examples, such as complex harmonic and cubic oscillators.

The talk is based on:

[1] S. Bögli, P. Siegl, and C. Tretter: *Approximations of spectra of Schrödinger operators with complex potentials on* $\mathbb{R}^d$, arXiv:1512.01826.

Gamow Shell Model: reaction observables

Nicolas Michel
National Superconducting Cyclotron Laboratory, Michigan State University
michel@nscl.msu.edu

The Gamow Shell Model (GSM), a configuration interaction model where many-body states are constructed from the one-body Berggren basis, bearing bound, resonant and scattering states, has been shown to be a powerful tool to describe the structure of drip-line nuclei as loosely bound and resonant many-body states can be efficiently calculated therein. The Cluster Orbital Shell Model (COSM), where nucleon coordinates are defined with respect to the center of mass of the core, allows to suppress center of mass excitations by directly working in a translationally invariant frame.

However, GSM cannot be used directly for the determination of reaction cross sections and observables. Indeed, as particle-emission channels cannot be identified in general in GSM, it is impossible to build scattering many-body states with well defined entrance and exit channels. To solve this problem, GSM has been extended to a reaction framework using the Resonating Group Method (RGM). In the RGM method, a basis of channels is constructed from target states and projectile states, which generate compound many-body basis states. Target states and projectile states are calculated in GSM, as they consist of bound or resonant eigenstates of the GSM Hamiltonian matrix. The Hamiltonian is then represented by a set of coupled integro-differential equations. Scattering many-body states are solutions of the Hamiltonian coupled-channel equations, from which reaction observables such as scattering and radiative capture cross sections can be calculated.

Due to the strong non-locality of coupling potentials, the coupled-channel equations of GSM-RGM are difficult to solve numerically. Consequently, a new method to solve coupled-channel equations has been introduced, where the coupled-channel Green function is represented by the Berggren basis. In particular, the complex-energy character of the Berggren basis states automatically provide proper outgoing asymptote to exit channels. The use of COSM coordinates also demands to carefully define the different projectile energies in different frames. While initially introduced with nucleon projectiles, GSM-RGM can be used as well with cluster projectiles. The main problem therein is to calculate projectiles with proper intrinsic and center of mass wave functions.

The GSM-RGM has been used to calculate the ^{18}Ne(p,p) and ^{14}O(p,p) reactions, where the proton-rich ^{19}Na and ^{15}F nuclei are unbound and thus are nuclei of choice for GSM and GSM-RGM. The ^{6}Li(p,p) scattering reaction, and ^{6}Li(p,γ)^{7}Be, ^{6}Li(n,γ)^{7}Li, ^{7}Be(p,γ)^{8}B and ^{7}Li(n,γ)^{8}Li radiative capture reactions, of astrophysical interest, will also be presented using GSM-RGM.

Dependence of Two-proton Radioactivity on Nuclear Pairing Models

Tomohiro Oishi[1−3], Markus Kortelainen[2,1], and Alessandro Pastore[4]
Helsinki Institute of Physics, University of Helsinki[1]*, Department of Physics, University of Jyvaskyla*[2]*, Research Center for Electron Photon Science, Tohoku University*[3]*, Department of Physics, University of York*[4]
tomohiro.t.oishi@jyu.fi[1]

Two-proton ($2p$) radioactive decay is one of the typical phenomena, in which the multi-particle quantum resonance plays an essential role. Its elucidation could provide an universal knowledge on the multi-particle quantum phenomena in various domains. Those include, *e.g.* the quantum entanglement, BCS-BEC crossover, and Efimov physics.

Thanks to the experimental developments, there has been a considerable accumulation of data for the $2p$-emitting nuclei. On the other side, however, theoretical studies have not been sufficient to clarify the relation between the $2p$ radioactivity and the nuclear force properties. For this purpose, several theoretical frameworks, in which the quantum resonance with non-perturbative interactions can be discussed, have been utilized. Those can be categorized into two classes: the time-independent, non-Hermitian framework and the time-dependent framework.

In this study, the sensitivity of two-proton emitting decays to the nuclear pairing correlation is discussed within a time-dependent three-body model [1, 2]. We focus on the ^{6}Be nucleus assuming $\alpha + p + p$ configuration, and its decay process is described as a time-evolution of the three-body resonance state. A noticeable model-dependence of two-proton decay width is found by utilizing schematic density-dependent contact (SDDC) and the finite-range Minnesota pairing models. The model-dependence with the SDDC pairing forces can be understood from the density distribution of the resonance state, which reflects a synergy of participating interactions. Our result suggests that two-proton decay width may be a suitable reference quantity to sophisticate the nuclear pairing model beyond the nucleon driplines [2].

References

[1] T. Oishi, K. Hagino, and H. Sagawa, Phys. Rev. C **90**, 034303 (2014).

[2] T. Oishi, M Kortelainen, and A. Pastore, arXiv: 1606.03111 (2016).

Qualitative analysis for certain classes of nonlinear oscillators and exploring the possibility of branched Hamiltonians

B.Bagchi
Department of Physics, School of Natural Sciences, Shiv Nadar University, Gautam Buddha Nagar, UP 201314, India
bbagchi123@gmail.com,Bijan.bagchi@snu.edu.in

We will carry out a systematic qualitative analysis of several quadratic schemes of generalized oscillators and perform a local analyses of the governing potentials. In this regard we intend to explore the possibility of branched Hamiltonians and consider a feasible quantization for such systems.

Non-Hermitian Hamiltonian and the Zeros of the Riemann Zeta Function

Dorje C. Brody
Department of Mathematics, Brunel University London, Uxbridge UB8 3PH, United Kingdom
Dorje.Brody@brunel.ac.uk

The Riemann hypothesis asserts that the nontrivial zeros of the Riemann zeta function should be of the form $\frac{1}{2} + \mathrm{i}\lambda_n$, where the set of numbers $\{\lambda_n\}$ are real. The so-called Hilbert-Pólya conjecture assumes that $\{\lambda_n\}$ should correspond to the eigenvalues of an operator $\hat{H}$ that is Hermitian. The discovery of such an operator $\hat{H}$, if it exists, thus amounts to providing a proof of the Riemann hypothesis. In 1999 Berry and Keating conjectured that such an operator $\hat{H}$ should correspond to a quantisation of the classical Hamiltonian $H = xp$. Since then, the Berry-Keating conjecture has been investigated intensely in the literature, but its validity has remained elusive up to now.

In this talk I will sketch the proof of the validity of the Berry-Keating conjecture. Specifically, I will derive the Hamiltonian, whose classical counterpart is $H = xp$, having the property that with a suitable boundary condition on its eigenstates, its the eigenvalues $\{\lambda_n\}$ corresponding to the nontrivial zeros of the Riemann zeta function. This Hamiltonian is not Hermitian, but is PT symmetric in a special way. A formal argument will be given for the associated metric operator and the formally 'Hermitian' counterpart Hamiltonian. Numerical evidence indicates that the eigenvalues of $\hat{H}$ are real, hinting at the validity of the Riemann hypothesis.

The talk is based on the work carried out in collaboration with Carl M. Bender (Washington University) and Markus Müller (University of Western Ontario).

Amphibious Complex Orbits and Dynamical Tunneling

Akira Shudo
Department of Physics, Tokyo Metropolitan University
shudo@tmu.ac.jp

Quantum tunneling takes place dynamically in mixed-types phase space where invariant tori and chaotic regions coexist. Complex semiclassical analysis reveals that a bunch of tunneling trajectories associated with chaos, or more precisely the Julia set in the complex plane, is involved in dynamical tunneling in mixed phase space. An important characteristic of complex orbits, which has been shown in a series of works by Bedford and Smillie, is that they have an amphibious character: they behave as regular orbits when the orbits stay in the regular region while as chaotic orbits when they wander in the chaotic domain. Exactly this character explains the emergence of "amphibious states", the states ignoring the underlying classical invariant structures.

Floquet topological phases of non-unitary quantum walks with $\mathcal{PT}$ symmetry

Hideaki Obuse[1], Ken Mochizuki[1], Dakyeong Kim[1], Norio Kawakami[2]
Department of Applied Physics, Hokkaido University[1],
Department of Physics, Kyoto University[2]
hideaki.obuse@eng.hokudai.ac.jp

A quantum walk, that is, a synthetic quantum system whose dynamics is described by a time-evolution operator, provides potential applications for quantum computing and information as well as quantum simulators. It is further interesting that the quantum walk possesses novel topological phases[1,2,3] akin to those of Floquet topological insulators, which are topological insulators driven by a time-periodic field. Recently, a non-unitary one-dimensional quantum walk dynamics associated with gain and loss is implemented in the fiber loops experiment [4]. The experiment shows that the quasienergy of the non-unitary time-evolution operator in the homogeneous system is kept to be real. This provides convincing evidence that the non-unitary time-evolution operator of this system possesses $\mathcal{PT}$ symmetry (combined parity and time-reversal symmetry). However, the definition of $\mathcal{PT}$ symmetry for the time-evolution operator has not yet been fully understood.

In this work, at first, we directly identify the $\mathcal{PT}$ symmetry operator and then verify $\mathcal{PT}$ symmetry of the time-evolution operator of the one-dimensional non-unitary quantum walk[5]. Taking this result into account, we then study Floquet topological phases of the $\mathcal{PT}$ symmetric non-unitary quantum walk[6]. We numerically observe that, by introducing position dependent parameters into the system by keeping $\mathcal{PT}$ symmetry of the time-evolution operator, localized states with zero and π quasienergies appear near the interface where the topological number varies from a value to the other one. We further confirm that the number of the localized states agree with the bulk-edge correspondence. We also find that only localized eigen states originating to Floquet topological phases break $\mathcal{PT}$ symmetry in the proper setup. This provides a way to observe highly intense probabilities of localized states originating to Floquet topological phases of the one-dimensional non-unitary quantum walk in actual experimental setups.

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[4] A. Regensburger, C. Bersch, M-A. Miri, G. Onishchukov, D. N. Christodoulides and U. Peschel, *Parity-time synthetic photonic lattices*, Nature **488** 167 (2012).
[5] K. Mochizuki, D. Kim, and H. Obuse, *Explicit definition of $\mathcal{PT}$ symmetry for nonunitary quantum walks with gain and loss*, Phys. Rev. A (accepted); arXiv:1603.05820.
[6] D. Kim, K. Mochizuki, N. Kawakami, and H. Obuse, (in preparation).

Pseudo $\mathcal{PT}$ Floquet theory

Mustapha Maamache
Laboratoire de physique quantique et systèmes dynamiques,
department of physics, Setif university, Setif 19000, Algeria
maamache_m@yahoo.fr, maamache@univ-setif.dz

We introduce the novel concept of the pseudo-parity-time (pseudo-$\mathcal{PT}$) symmetry in periodically systems and precisely in Floquet operator. We analyze the dynamics of time-periodic $\mathcal{PT}$-symmetric Hamiltonians by means the PT-symmetric extension of the Moore and Stedman decomposition of the non-unitary Floquet operator. We establish that the stablility of the dynamics occur when the $\mathcal{PT}$-symmetry of the time-independent operator L which define the Floquet operator $U(\tau)=e^{-iL\tau}$, where τ is the period of the $\mathcal{PT}$-symmetric Hamiltonian, is unbroken. This situation correspond to the real quasienergies ϵ_n. Nevertheless, when the $\mathcal{PT}$-symmetry of L is broken, which correspond to the complex conjugates quasienergies ϵ_n, an instable dynamics arise. As an illustrative example, we studied in greater detail a new example of a the periodically driven oscillators in the $\mathcal{PT}$-symmetric harmonic potential.

Recent Developments about the Physics of Exceptional Points

W Dieter Heiss
Department of Physics, University of Stellenbosch, South Africa
dieter@physics.sun.ac.za

A survey is given about the physics of exceptional points. Particular emphasis is given to more recent theoretical and experimental results.

Time arrow and complex spectral analysis of Liouvillian and Hamiltonian dynamics in terms of non-Hermitian operators

Tomio Petrosky
Center for Studies in Statistical Mechanics and Complex Systems, The University of Texas at Austin
petrosky@physics.utexas.edu

General overview review of recent results of the complex spectral analysis of the Liouvillian dynamics as well as the Hamiltonian dynamics developed by Austin-Osaka group is presented. From the microscopic fundamental laws of physics, time arrow with a broken-time-symmetry can be deduced through the resonance singularities in the so-called small-denominator for the open system. Due to the resonance singularities, the Hermitian generator of motion in the Hilbert space leads to non-Hermitian effective Liouvillian and/or Hamiltonian with complex eigenvalues in the extended function space. These operators share the same eigenvalue with the original Liouvillian and/or Hamiltonian. The imaginary parts of the eigenvalues give the transport coefficients in irreversible processes. In this talk the irreversible process associated to the Jordan block and the so-called exceptional points that have no counter part in Hermitian dynamics will be discussed. Some open problems in the mathematical formation will be also stated.

Jordan block of the total Hamiltonian in the extended Hilbert space for open quantum systems

Kazuki Kanki
Department of Physical Science, Osaka Prefecture University
kanki@p.s.osakafu-u.ac.jp[1]

We show that the Hamiltonian for an open quantum system at an exceptional point is represented by a Jordan block in a subspace spanned with basis vectors which are outside of the Hilbert space.[1] We describe how to obtain a Jordan basis which consists of dual eigenvectors and associated dual pseudoeigenvectors on the basis of the Feshbach projection method. In this method the complex eigenvalue problem of the total Hamiltonian is reduced to that of the effective Hamiltonian which depends on the eigenvalue itself. Because of the eigenvalue dependence, the effective Hamiltonian cannot be represented as a matrix having a Jordan block, in contrast to the case of a phenomenological effective Hamiltonian.

Actually the Jordan block form is not restricted to exceptional points, but even away from exceptional points an operator can be represented by a matrix having a Jordan block.[2] We give a Jordan block away from an exceptional point by making a Jordan basis in a subspace spanned by two eigenstates which coalesce at an exceptional point, and show that this generalized Jordan block connects continuously to the Jordan block just at the exceptional point.

[1] T. Petrosky, I. Prigogine, and S. Tasaki, Physica A **173**, 175 (1991).
[2] K. Hashimoto, K. Kanki, H. Hayakawa, and T. Petrosky, Prog. Theor. Exp. Phys. **2015**, 023A02 (2015).

Characteristic dynamics near two coalescing eigenvalues incorporating continuum threshold effects

Savannah Garmon[1], Gonzalo Ordonez[2]
Department of Physical Science, Osaka Prefecture University[1], *Department of Physics and Astronomy, Butler University*[2]
sgarmon@p.s.osakafu-u.ac.jp[1], gordonez@butler.edu[2]

The survival probability $P(t)$ near two coalescing eigenstates at an exceptional point has been reported in the literature as following $P(t) \sim t^2 e^{-\Gamma t}$, in which Γ is the decay width of the coalesced eigenvalue; this result has been confirmed in a microwave cavity experiment [1]. However, the theoretical analysis used to obtain this result usually employs a heuristic finite Hamiltonian that describes only the two modes coalescing in the vicinity of the exceptional point. In this work, we emphasize that this ad hoc approach washes out the details of the continuum and, in particular, ignores the existence of the continuum threshold; as a result it does not correctly describe the time evolution near the exceptional point on all time scales and completely fails in some cases.

To report our results, we divide the exceptional points in Hermitian open quantum systems into two cases: at an EP2A two virtual bound states coalesce before forming a resonance, anti-resonance pair with complex conjugate eigenvalues, while at an EP2B two resonances coalesce before forming two different resonances [2]. We use two simple models to study the EP2A and EP2B as representative cases. For the EP2A we point out that the evolution is non-exponential on all timescales and that the influence of the continuum threshold may be quite significant [3]. When the EP2A appears very near the threshold we obtain the novel evolution $P(t) \sim 1 - C_1\sqrt{t} + D_1 t$ on intermediate timescales, while further away the parabolic decay (Zeno dynamics) on short timescales is very prominent. For the EP2B, which is the case studied in the microwave cavity experiment, we find the survival probability evolves as $P(t) \sim (1 - C_2 t + D_2 t^2)e^{-\Gamma t}$ on intermediate timescales. In either case, an inverse power law decay controls the system dynamics on long time scales.

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Resonances for perturbed Dirac operators

M. Melgaard
School of Mathematical and Physical Sciences, University of Sussex, Great Britain
m.melgaard@sussex.ac.uk

We prove the existence of quantum resonances of the three-dimensional Dirac operator perturbed by smooth, bounded and real-valued scalar potentials V decaying like $\langle x \rangle^{-\delta}$ at infinity for some $\delta > 0$. By studying analytic singularities of a certain distribution related to V and by combining two trace formulas, we prove that the perturbed Dirac operators possess resonances near $\sup V + 1$ and $\inf V - 1$. Furthermore, for smooth compactly supported Hermitian matrix $V(x)$, we establish a global Poisson wave trace formula.

Nonhermitian Topological Phases

Masatoshi Sato
Yukawa Institute for Theoretical Physics, Kyoto University
msato@yukawa.kyoto-u.ac.jp

Topological stability of the edge states is investigated for non-Hermitian systems. We examine two classes of non-Hermitian Hamiltonians supporting real bulk eigenenergies in weak non-Hermiticity: SU(1,1) and SO(3,2) Hamiltonians. As an SU(1,1) Hamiltonian, the tight-binding model on the honeycomb lattice with imaginary on-site potentials is examined. Edge states with ReE=0 and their topological stability are discussed by the winding number and the index theorem, based on the pseudo-anti-Hermiticity of the system. As a higher symmetric generalization of SU(1,1) Hamiltonians, we also consider SO(3,2) models. We investigate non-Hermitian generalization of the Luttinger Hamiltonian on the square lattice, and that of the Kane-Mele model on the honeycomb lattice, respectively. Using the generalized Kramers theorem for the time-reversal operator Theta with $\Theta^2 = +1$, we introduce a time-reversal invariant Chern number from which topological stability of gapless edge modes is argued.

$\mathcal{CPT}$-consereved effective mass Hamiltonians through first and higher order charge operator $\mathcal{C}$ in a supersymmetric framework

Asish Ganguly
Department of Mathematics, Indian Institute of Technology Kharagpur, Kharagpur 721302, India
aganguly@maths.iitkgp.ernet.in, gangulyasish@rediffmail.com

The features of a generalized position-dependent mass Hamiltonian H_m in a supersymmetric framework in which the constraints of pseudo-Hermiticity and $\mathcal{CPT}$ are naturally embedded. Different representations of the charge operator are considered that lead to new mass-deformed superpotentials $\mathcal{W}_m(x)$ which are inherently $\mathcal{PT}$-symmetric. The qualitative spectral behavior of H_m is studied and several interesting consequences are noted.

Aharanov-Anandan Phase in Non-unitary Cyclic Dynamics

Qing-hai Wang
Department of Physics, National University of Singapore
qhwang@nus.edu.sg

Non-Hermitian systems describe a wide class of systems in the classical domain and can be also regarded as extensions of conventional quantum systems. Their spectral and dynamical features have attracted considerable interests. In this talk, I will expose rich physics in non-unitary cyclic time evolution of non-Hermitian systems, using Aharonov-Anandan (AA) phase as a quantitative diagnostic tool. I will first clear much confusion in the literature by showing that the AA phase is always real, and a previous expression for AA phase, once slightly modified, can equally apply to non-Hermitian systems. I will then analyze AA phase in two periodically driven non-Hermitian models. In the slow-driving limit, the AA phase reduces to the Berry phase in the first case, but oscillates violently and does not approach any limit in the second case. The rich geometrical features of non-unitary dynamics are thus seen to be a largely unexplored but fruitful topic for future theoretical and experimental studies.

$\mathcal{PT}-$symmetry, flip symmetry and Hilbert-Schmidt Lie groups in Krein spaces

Uwe Günther
Helmholtz-Center Dresden-Rossendorf, Dresden, Germany
u.guenther@hzdr.de

The first part of the talk is devoted to flip-symmetric representations of finite-dimensional $\mathcal{PT}-$symmetric matrices and their technical usefulness in concrete calculations and structure analyses.

In the second part of the talk, earlier results on $\mathcal{PT}-$symmetry related Lie algebra structures, Cartan decompositions and Lie triple systems for finite-dimensional matrix algebras are extended to infinite-dimensional setups of Hilbert-Schmidt Lie algebras and Hilbert-Schmidt Lie groups acting in Krein spaces. The specific structure properties of these algebras and groups are discussed and possible applications briefly sketched.

This is work in collaboration with Jia-wen Deng and Qing-hai Wang.

Non-Hermitian degeneracies in the Lieb-Liniger model and exotic quantum holonomy

Atushi Tanaka
Tokyo Metropolitan University
tanaka-atushi@tmu.ac.jp

In thermodynamics, adiabatic cycles are trivial as they induce no change. On the contrary, quantum mechanical systems may be changed by adiabatic cycles. A famous example is the geometric phase. Even the adiabatic cycles may induce excitation of quantum mechanical systems. The latter is referred to as the exotic quantum holonomy [1]. It has been shown that the exotic quantum holonomy has an intimate correspondence with the degeneracy points that appear in non-Hermitian quantum theory [2]. We also note that the covering space structure plays a crucial role in the topological formulation of the exotic quantum holonomy [1] as well as the theory for the geometric phase in non-Hermitian systems [3]

In this talk, I will explain an interplay of the exotic quantum holonomy and exceptional points in one-dimensional Bose systems [4,5]. In particular, we examine an adiabatic cycle that starts from the free system and goes through Tonks-Girardeau and super-Tonks-Girardeau regimes and comes back to the free system. In the Hermitian side, it is shown that the exotic holonomy occurs with an arbitrary number of Bose particles, where the role of conserved quantities is emphasized. The analytic continuation of the adiabatic cycles that induces the exotic quantum holonomy in two-body case clarifies the role of non-Hermitian degeneracy points of the Lieb-Liniger model [6].

This is a joint work with Nobuhiro Yonezawa and Taksu Cheon.

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Entanglement of spin 1/2 particles with PT symmetric Hamiltonian in a Schwarchild-De Sitter space-time

N. Mebarki , A. Mohadi and M. Boussahel
Laboratoire de Physique Mathematique et Subatomique, Mentouri University, Constantine 1, Algeria
nnmebarki@yahoo.fr

Entanglement of Pauli spinors in a Schwarchild-De Sitter space-time with PT symmetric Hamiltonian is studied. It is shown that even if the initial state is non entangled, a non vanishing ebit of entanglement can be generated during a time evolution of the wave packet. The various spin configurations of the matrix density of singlet and triplet states are derived and some results are discussed.

Time-delayed PT-symmetric systems: theory and experiments

Yogesh N. Joglekar, Joseph Suelzer, and Gautam Vemuri
Indiana University Purdue University Indianapolis (IUPUI).
yojoglek@iupui.edu

Open classical or quantum systems with balanced gain and loss have attracted a lot of attention in recent years. Such systems are described by an effective, non-Hermitian Hamiltonian that is invariant under combined parity and time reversal operations. A key feature of such systems is the PT-symmetry breaking transition, which occurs when then the strength of the non-Hermiticity is larger than the energy scale set by the Hermitian part of the Hamiltonian. Physically, this transition leads to a state that is far removed from equilibrium because the local excesses or deficits, created by the gain and the loss potentials respectively, are not rapidly neutralized. All of these models are governed by first or second order time-differential equations, and thus local in time.

Here I will introduce PT-symmetric systems with a time delay, which physically encodes the finite speed of communication between the gain and the loss, and show how they can be realized in coupled semiconductor lasers. I will show that the simplest model of time-delayed PT-symmetric system - a two-site model with balanced gain and loss - shows a rich phase diagram with multiple PT-symmetric and broken regions. I will discuss the similarities and differences between PT-symmetric systems with time delay and the PT-symmetric Rabi problem. I will then present experimental results for the time-delayed PT-symmetric system.

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Multidimensional Generalization of Transfer Matrices, Transmission Reciprocity, and Unidirectional Invisibility

Farhang Loran[1], Ali Mostafazadeh[2]
Department of Physics, Isfahan University of Technology, Iran[1],
Departments of Mathematics and Physics, Koç University, Turkey[2]
loran@cc.iut.ac.ir[1], amostafazadeh@ku.edu.tr[2]

We outline an alternative to the S-matrix formulation of scattering theory in arbitrary dimensions that relies on a multidimensional generalization of the notion of transfer matrix and makes use of certain pseudo-Hermitian and pseudo-normal Hamiltonian operators. This enables us to provide an exact treatment of delta function potentials in two and three dimensions, establish a genuinely multidimensional generalization of the Lorentz reciprocity principle, and extend the concept of unidirectional invisibility to two and three dimensions. We show that the multidimensional reciprocity principle does not prohibit nonreciprocal transmission, thus lifting a major obstruction to devise optical and acoustic diodes using linear material. We give a general method of constructing unidirectionally invisible potentials in two and three dimensions that can support nonreciprocal transmission. As a concrete physical application of our findings, we construct an active optical wire with a rectangular cross-section that is invisible from the right but displays nontrivial reflection and transmission from the left.

References: Phys. Rev. A 93, 042707 (2016) & Preprint arXiv:1605.01225, to appear in Proc. R. Soc. A.

Some developments in few-particle scattering problem based on complex-scaling method

Rimantas Lazauskas[1]
IPHC, IN2P3-CNRS/Univ. Louis Pasteur BP 28, F-67037 Strasbourg Cedex 2, France[1]
`rimantas.lazauskas@iphc.cnrs.fr`[1]

The solution of the scattering problem in configuration space is a very difficult task both from formal (theoretical) as well as computational points of view. The principal difficulties arise from the complex asymptotic behavior of the system wave function, which may be result of either the presence of multiple scattering channels or the systems breakup into three or more clusters. One is obliged to seek for the exact methods enabling to treat the multiparticle scattering problem by avoiding the explicit treatment of the systems wave-function at the boundaries.

In particular complex-scaling method, proposed in the late sixties by Nuttal and Cohen [1], offers very accurate and elegant formalism to treat diverse scattering problems for short range potentials [2]. Within the last few-years [3, 4, 5] I have applied this method in handling very different scattering problems: 2-body collisions including Coulomb interaction, Optical potentials; scattering including the 3-body break-up for real and Optical short-ranged interactions; 3-body and 4-body scattering for the systems, where two-particles (clusters) are charged; 3-body break-up amplitude for n-d as well as p-d scattering. Finally, I have demonstrated that the conventional smooth complex scaling technique might be also used in describing collisions in pure Coulombic 3-body systems. These late achievements will be overviewed and discussed.

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Scattering Functions in terms of an effective non-hermitean hamiltonian

G.A. Luna-Acosta[1], A.A. Fernández-Marín[2]
Instituto de Física, Benemérita Universidad Autónoma de Puebla, Apartado Postal J-48, Puebla 72570, Mexico[1], *Instituto de Ciencias Físicas, Universidad Nacional Autónoma de México, A.P. 48-3, 62251 Cuernavaca, Morelos, Mexicoo*[2]
glunaacosta@gmail.com[1], antoniofdzm@gmail.com[2]

For one dimensional scattering off finite range potentias we derive expressions for time delay, effective scattering dwelling distance, trapping probability and cross section in terms of the Reaction Matrix and an associated non-hermitian effective hamiltonian . We show the procedure to calculate poles using the effective hamiltonian. We also compare one-level and one-pole calculations with exact numerics for representative systems.

Broadband Coherent Perfect Absorption

Bhabani Prasad Mandal [1]
Department of Physics, Institute of Science, Banaras Hindu University, Varanasi-221005 [1]
bhabani.mandal@gmail.com[1]

Coherent Perfect Absorption (CPA) is very important characteristic of complex potentials with the potential of technological applications. However CPA has been reported only for certain discrete incident energy so far. In this work we onsider the most general non-Hermitian Hulthen potential to study the scattering of spin-less relativistic particles. The conditions for CC, SS and CPA are obtained analytically for this potential. We show that almost total absorption occurs for entire range of incidence energy for certain parametric ranges of the potential and hence term this as 'black potential'. Time reversed of the same potential shows perfect emission for the entire range of particle energy. We also present the classical analog of this potential in terms of waveguide cross section.

Quantization of β-Fermi-Pasta-Ulam Lattice with Nearest and Next-nearest Neighbour Interactions

Aniruddha Kibey[1], Rupali Sonone[2], Bishwajyoti Dey[3], J. Chris Eilbeck[4]
Department of Physics, University of Pune, Pune - 411007, India[1], *Department of Physics, University of Pune, Pune - 411007, India*[2], *Department of Physics, University of Pune, Pune - 411007*[3], *Department of Mathematics and Maxwell Institute, Heriot-Watt University, Riccarton, Edinburgh, EH14 4AS, UK*[4]
aniruddha@cms.unipune.ac.in[1], vaidehisonone@gmail.com[2]

We quantize the β-Fermi-Pasta-Ulam (FPU) model with nearest and next-nearest neighbor interactions using a number conserving approximation and a numerically exact diagonalization method. Our numerical mean field bi-phonon spectrum shows excellent agreement with the analytic mean field results of Ivić and Tsironis (Z. Ivić and G. Tsironis. Physica D, 216, 200, 2006), except for the wave vector at the midpoint of the Brillouin zone. We then relax the mean field approximation and calculate the eigenvalue spectrum of the full Hamiltonian. We show the existence of multi-phonon bound states and analyze the properties of these states by varying the system parameters. From the calculation of the spatial correlation function we then show that these multi-phonon bound states are particle like states with finite spatial correlation. Accordingly we identify these multi-phonon bound states as the quantum equivalent of the breather solutions of the corresponding classical FPU model. The four-phonon spectrum of the system is then obtained and its properties are studied. We then generalize the study to an extended range interaction and consider the quantization of the β-FPU model with next-nearest-neighbor interactions. We analyze the effect of the next-nearest-neighbor interactions on the eigenvalue spectrum and the correlation functions of the system.

Reference: A. Kibey, R. L. Sonone, B. Dey, and J. C. Eilbeck, Fermi-Pasta-Ulam lattice with nearest and next-nearest neighbour interactions, Physica D **294**, 43 (2015).

Solutions of the Schrödinger equation in terms of the Heun functions

Artur Ishkhanyan
Institute for Physical Research, NAS of Armenia, Ashtarak 0203, Armenia
aishkhanyan@gmail.com

We review the cases for which the Schrödinger equation is solved in terms of the general and confluent Heun functions. We present the possible choices for the coordinate transformation that provide energy-independent potentials that are proportional to an energy-independent continuous parameter and have a shape independent of that parameter. In contrast to the hypergeometric case, no Heun potential can in general be transformed into another one by specifications of the involved parameters.

We show that there exist in total 29 independent Heun potentials. There are eleven independent potentials that admit the solution in terms of the general Heun functions, for nine independent seven-parametric potentials the solution is given in terms of the confluent Heun functions, there are three independent double-confluent and five independent bi-confluent Heun potentials (the six-parametric Lamieux-Bose potentials), and one tri-confluent Heun potential (the general five-parametric quartic oscillator).

There are several independent potentials that present distinct generalizations of either a hypergeometric or a confluent hypergeometric classical potential, some potentials possess sub-cases of both hypergeometric types, and others possess particular conditionally integrable ordinary or confluent hypergeometric sub-potentials. We present several examples of explicit solutions for the latter potentials.

We show that there exist other exactly or conditionally integrable sub-potentials the solution for which is written in terms of simpler special functions. However, these are solutions of different structure. For instance, there are sub-potentials for which each of the two fundamental solutions of the Schrödinger equation is written in terms of irreducible combinations of hypergeometric functions. Several such potentials are derived with the use of deformed Heun equations. A complementary approach is the termination of the hypergeometric series expansions of the solutions of the Heun equations.

$\mathcal{PT}$-symmetry and integrability as reality conditions for complex solitons

Andreas Fring
Department of Mathematics, City University London, Northampton Square, London EC1V 0HB, UK
a.fring@city.ac.uk

We report complex $\mathcal{PT}$-symmetric multi-soliton solutions to the Korteweg de-Vries, modified Korteweg de-Vries and complex sine-Gordon equation. The solutions are obtainable from Hirota's direct method or multiple Bäcklund transformations. We show that complex solitons possess the new feature of allowing to construct compound solutions that asymptotically contain one-soliton solutions, with each of them possessing the same amount of finite real energy. We demonstrate how these solutions originate from degenerate energy solutions of the Schrödinger equation, which is technically achieved by the application of Darboux-Crum transformations involving Jordan states with suitable regularizing shifts. We compute the time-delays resulting in a multi-soliton scattering and argue that $\mathcal{PT}$-symmetry together with integrabilty guarantees the reality of all conserved charges.

Density of Eigenvalues in a generalized Joglekar-Karr Quasi-Hermitian Matrix Model

Joshua Feinberg
Physics Department, University of Haifa at Oranim and Technion
joshua@physics.technion.ac.il

We discuss a slight generalization of the model of random quasi-hermitian matrices introduced by Joglekar and Karr several years ago in Phys. Rev. E83 (2011) 031122. This generalized ensemble is comprised of $N \times N$ matrices $M = AF$, where A is a complex-hermitian matrix drawn from the $U(N)$-invariant probability distribution $P(A) = \frac{1}{Z}\exp[-N\mathrm{Tr}V(A)]$ (Z is a normalization factor and $V(A)$ is typically some polynomial), and F is a strictly-positive hermitian matrix. (In the original Joglekar-Karr model, A was taken to be a Gaussian random matrix.) With no loss of generality (due to $U(N)$ symmetry), F can be taken to be diagonal. The matrix M is non-hermitian, of course, but can be brought to a hermitian form $H = \sqrt{F}A\sqrt{F}$ by means of a similarity transformation. All its eigenvalues are therefore real. Bringing some powerful tools of Random Matrix Theory to bear, we obtain, in the large-N limit, explicit analytical expressions for the density of eigenvalues of M.

Non-Hermitian Localization and Delocalization in the Generalized Feinberg-Zee Model

Naomichi Hatano
Institute of Industrial Science, University of Tokyo
hatano@iis.u-tokyo.ac.jp

We consider the following non-Hermitian tight-binding model [1]:

$$H = \sum_{x=1}^{L} \left(s_x |x+1\rangle\langle x| + t_x |x\rangle\langle x+1| \right), \tag{1}$$

where we choose the hopping elements to the right, $\{s_x\}$, and to the left, $\{t_x\}$, independently randomly from ± 1. We also impose the periodic boundary condition. Such an $L \times L$ matrix can represent, for example, a neural network in which the synapse connection from the xth neuron to the $x+1$th one and the connection in the opposite direction may be independently excitatory or inhibitory. The model has been introduced by Feinberg and Zee [2] in the context of random matrix theory and is known to show a fractal-like spectrum.

In order to know the localization nature of the eigenvectors of the Feinberg-Zee model, we introduce a non-Hermitian gauge field g in the spirit of the Hatano-Nelson model [3], as follows:

$$H = \sum_{x=1}^{L} \left(e^{g} s_x |x+1\rangle\langle x| + e^{-g} t_x |x\rangle\langle x+1| \right). \tag{2}$$

We found that a hole emerges in the middle of the spectrum and grows as we increase g [1]. We argue that the eigenstates on the rim of the hole for g has the inverse localization length equals to g. This is in consistent with a recent numerical calculation [4] based on the Chebyshev-polynomial expansion of the inverse localization length.

This work is based on the collaboration with Ariel Amir and David R. Nelson [1].

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Pseudo-Hermitian ensemble of random Gaussian matrices

G. Marinello[1], M. P. Pato[2]
Instituto de Física, Rua do Matão 1371, Universidade de São Paulo, 05508-090, São Paulo, S.P., Brazil
gamasosa@gmail.com[1], mpato@if.usp.br[2]

It is shown how pseudo-Hermiticity, a necessary condition satisfied by operators of PT symmetric systems can be introduced in the orthogonal, unitary and sympletic Gaussian classes of random matrix theory[1]. The method consists in the introduction of projection operators to perform a decomposition of the matrices into diagonal and off-diagonal blocks. It is shown that the form of the off-diagonal blocks that couple the diagonal ones is determined by imposing the pseudo-Hermiticity condition. The joint density distribution of matrix elements is analitically derived and, from it, an ansatz is proposed that describes the localization of the complex eigenvalues inside an ellipse determined by the strength of the coupling. The ansatz is confirmed by numerical simulations. In the model, the pseudo-Hermitian nature of each of the matrices' blocks may vary, and the connection between them is controlled by coupling parameters in the off-diagonal blocks as well as the size of the blocks. This leads to a model that describes transitions from real eigenvalues to a situation in which, apart from a residual number, the eigenvalues are complex conjugate. It is also shown how the block decomposition relates to the basic PT symmetric complex Hamiltonians[2]. The present results extend previous recent ones obtained with ensemble of tridiagonal matrices[3, 4, 5].

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Hermitian, Pseudo-Hermitian and PT-symmetric Hamiltonians on graphs and singular spaces. Geometrical, asymptotical and statistical properties of the time-dependent Schrödinger equation.

Andrei Shafarevich
M.V. Lomonosov Moscow State University
shafarev@yahoo.com

We study time dependent Scrödinger equations on graphs and singular spaces; the latter are obtained from graphs via replacing vertices by 2D or 3D manifolds. The corresponding Hamiltonians are defined with the help of extension theory and supposed to be either Hermitian or Pseudo-Hermitian or PT-symmetric. We describe the evolution of the localized (squeezed) initial state. Semi-classical behavior of the wave function appears to be connected with the structure of geodesics on the manifolds. Calculation of the number of wave packets can be reduced to well known problems of the analytic number theory.

On the pseudo-Hermitian invariant method for the time-dependent Non-Hermitian Hamiltonians

Boubekeur Khantoul[1], Abdelhafid Bounames[1], Mustapha Maamache[2]
Department of Physics, University of Jijel, Algeria.[1],
Department of Physics, University of Setif 1, Algeria.[2]
a.bounames@gmail.com[1], bounames@univ-jijel.dz[1]

A consistent pseudo-Hermitian invariant operator theory of quantum mechanics can be built on a complex time-dependent non-Hermitian Hamiltonian. We use the pseudo-Hermitian invariant operator in order to study the general time-dependent problems in non-Hermitian quantum mechanics, e.g., those with a time-dependent non Hermitian Hamiltonian and with a time-dependent metric. The pseudo-Hermitian invariant has positive real eigenvalues and implies that the dynamics is governed by unitary time evolution. As a consequence, the phase associated with the non-Hermitian evolution is real. This work is not in contradiction with conventional pseudo quantum mechanics studied recently but is rather a complex generalization of it. The harmonic oscillator with a time-dependent frequency under the action of a complex time-dependent linear potential is considered as an illustrative example.

Different types of conjugation operations and hidden symmetry for matrix non-Hermitian Hamiltonians

Andrey Sokolov
Saint-Petersburg State University
avs_avs@rambler.ru, andrey.sokolov@spbu.ru

It is known [A. Andrianov and A.S., NP B **660** (2003) 25 and NP B **773** (2007) 107] that in the scalar case one can construct with the help of transposing polynomial algebra of supersymmetry from two non-Hermitian, in general, Hamiltonians and arbitrary differential operator that intertwines these Hamiltonians. Moreover, in the case of extended algebra of supersymmetry one can build with the use of transposing antisymmetric hidden symmetry operators for both intertwined Hamiltonians and these operators possess by interesting properties closely related with spectral properties of these Hamiltonians. In the matrix case transposing cannot, in general, to help find for a given intertwining operator an operator that intertwines the same Hamiltonians in the opposite direction since these Hamiltonians are not symmetric, in general. In the report we seek for such generalization of transposing that allows us to construct polynomial algebra of supersymmetry from two non-Hermitian, in general, matrix Hamiltonians and arbitrary matrix differential operator that intertwines these Hamiltonians. As well we consider properties of hidden symmetry operators built in the case of extended algebra with the help of the indicated above generalized operation.

Non-Hermitian versions of the Heisenberg and interaction pictures

Miloslav Znojil
Nuclear Physics Institute of the Czech Academy of Sciences, 250 68 Řež, Czech Republic
znojil@ujf.cas.cz

Besides the standard non-Hermitian version of quantum mechanics of unitary evolution presented in Schrödinger picture (and often combined with the heuristically productive concept of PT-symmetry), we shall show that and how one can also describe unitary evolution using various non-Hermitian generalizations of the Heisenberg and interaction (*a.k.a.* Dirac's) pictures. Several elementary illustrative examples will be provided and several fundamental concepts (like the Haag's or Stone's *no-go* theorems) will be discussed.

Part II: Poster Presentations

Exact spectral densities of non-Hermitian noise-plus-structure random matrices

Jacek Grela[1], Thomas Guhr[2]
Jagiellonian University, Krakow[1]*, University of Duisburg-Essen, Duisburg*[2]
jacekgrela@gmail.com[1], thomas.guhr@uni-due.de[2]

We use supersymmetry to calculate exact spectral densities for a class of non-Hermitian complex random matrix models having the form $M = S + LXR$, where X is a random noise part X and S, L, R are fixed structure parts. This is a certain version of the "external field" random matrix models. We found two-fold integral formulas for arbitrary structural matrices. We investigate some special cases in detail and carry out numerical simulations. The presence or absence of a normality condition on S leads to a qualitatively different behaviour of the eigenvalue densities.

Relevant paper: http://arxiv.org/abs/1605.01159.

Dynamics of interaction of Peregrine solitons in the continuous nonlocal Schrödinger system with parity-time symmetry

Samit Kumar Gupta[1], Amarendra K. Sarma[2]
Indian Institute of Technology Guwahati[1], Indian Institute of Technology Guwahati[2]
samit.kumar.gupta@gmail.com[1]

Recently Ablowitz and Musslimani [*Phys. Rev. Lett.* ***110****, 064105 (2013)*] have proposed an integrable nonlocal nonlinear Schrödinger equation model where the standard third order nonlinearity $|\psi(x,z)|^2\psi(x,z)$ is replaced by its Parity-time-symmetric form: $\psi(x,z)\psi^*(-x,z)\psi(x,z)$. This equation admits Peregrine soliton solutions. This model also supports Peregrine rogue wave in the broken PT-phase for a single initial Peregrine soliton excitation. In this work, we report a numerical investigation of the interaction of two first order Peregrine solitons in both in-phase and out-of-phase conditions. We find that for the in-phase condition, in the unbroken PT-phase, the model yields the KM soliton breather when the transverse shift is small. On the other hand, in the broekn PT-phase it yields instability behaviors. In the out-of-phase condition, the model results in oblique propagation dynamics of the optical fields.

Tunneling in nearly integrable systems with a non-hermitian perturbation

Yasutaka Hanada[1], Akira Shudo[2] and Kensuke S. Ikeda[3]
Tokyo Metropolitan University[1,2], *Ritsumeikan University*[3]
`hanada@tmu.ac.jp`[1], `shudo@tmu.ac.jp`[2], `ahoo@ike-dyn.ritsumei.ac.jp`[3]

We study the tunneling effect in nearly integrable systems with a non-hermitian perturbation. In the integrable system such as a particle moving in the one-dimensional double well potential, the energy splitting ΔE caused by quantum tunneling is evaluated as

$$\Delta E \underset{\hbar \to 0}{\sim} e^{-S/\hbar}, \tag{3}$$

by the semiclassical (WKB) approximation, where the action S is determined by the classical quantity [1, 2]. On the other hand, in the systems under the periodic perturbation, the corresponding classical system becomes non-integrable. If one plots the energy splitting as a function of $1/\hbar$, it exhibits persistent enhancement from the prediction (1) accompanying spikes.

The spike can be interpreted as energetic resonance with excited states by photon absorption in the language of quantum dynamics, but it may be reinterpreted by the language of classical dynamics. The theory of resonance-assisted tunneling (RAT) have discussed the relation between the appearance of the spikes and the classical non-linear resonances, and then it claimed that the classical non-linear resonances create a bunch of spikes, which brings the enhancement of tunneling probability [3, 4].

The appearance of the spikes in the energy splitting has been considered as the origin of the enhancement of tunneling probability, but to make clear this issue, we introduce a weak non-hermitian perturbation which pushes the resonant states to the complex domain. By applying this perturbation, we found the spikes and the persistent enhancement have the different origin, and it was unveiled that the staircase-like structure is hidden in the energy splitting curve as a function of $1/\hbar$ [5].

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Quantum centrality testing on directed graphs via PT-symmetric quantum walks

Josh Izaac[1], Jingbo Wang[2], Xiaosong Ma[3]
School of Physics, The University of Western Australia, Crawley WA 6009[1,2]
School of Physics, Nanjing University, Nanjing, 210093, China[3]
josh.izaac@uwa.edu.au[1], jingbo.wang@uwa.edu.au[2], xiaosong.ma@nju.edu.cn[3]

Quantum walks are an important tool in the field of quantum information theory. Indeed as a method of universal quantum computation, they have motivated the creation of quantum algorithms that are faster and more efficient than their classical analogues, and provided a vital link between quantum computation and modeling complex quantum dynamical systems (for example, photosynthesis). However, one disadvantage of the quantum walk is the imposition of unitarity, due to the quantum nature of the walkers. As such, the conventional quantum walk is unable to model or analyze directed network structures, without either a) resulting in non-unitary dynamics, or b) modifying the framework. This serves as a particular hindrance in extending established quantum algorithms (e.g. quantum search, centrality measures, graph isomorphism) and quantum dynamical models to systems with direction/biased potentials (such as transport of electrons or excitons).

One such solution to this problem lies in the field of PT-symmetry, which offers the capability to perform quantum walks on directed graphs with non-Hermitian Hamiltonians whilst preserving the norm. In this presentation, we formalize a rigorous framework for continuous-time quantum walkers (CTQWs) on pseudo-Hermitian directed graph structures. This is then extended to the cases of multi-particle quantum walks and interdependent networks, before being utilized to measure vertex centrality in various small graphs – resulting in strong agreement with the classical PageRank algorithm, and in some cases even distinguishing vertex equivalence classes not identified by PageRank. Unlike previous quantum centrality measures, for a graph of N vertices this algorithm requires a Hilbert space of dimension N (compare this to the Szegedy quantum walk PageRank algorithm, which requires a N^2 dimensional Hilbert space), and without the classical decoherence required for open quantum systems. Furthermore, we show that this formalism is equivalent to considering an undirected, yet weighted, complete graph with self-loops, providing a structural interpretation that may lead to simple experimental implementation.

Dirichlet spectrum of the complex PT-symmetric Bender-Boettcher potential: $\mathbf{V(x) = -(ix)^{\nu}}$

Zafar Ahmed[1], Sachin Kumar[2], Dhruv Sharma[3]
Nuclear Physics Division, Bhabha Atomic Research Centre, Trombay, Mumbai, 400085, India[1]
Theoretical Physics Section, Bhabha Atomic Research Centre, Trombay, Mumbai, 400085, India[2]
Department of Physics, National Institute of Technology, Rourkela, 769008, India[3]
zahmed@barc.gov.in[1], sachinv@barc.gov.in[2], sharmadhruv@gmail.com[3]

Normally, in quantum mechanics discrete spectrum of a one dimensional potential may consist of real eigenvalues corresponding to bound states or perfect transmission scattering states and complex eigenvalues corresponding to resonant states. For bound states one usually demands Dirichlet boundary condition on the wave function as $\psi(\pm\infty) = 0$ to extract the real eigenvalues. Since the beginning of the PT-symmetric quantum mechanics, the concept of eigenvalues in wedges of the complex x-plane has been professed and utilized to find the real discrete spectrum of the paradigm model: $V_{BB}(x) = -(ix)^{\nu}$ of Bender and Boettcher by varying ν. However, one would like to know the real spectrum of $V_{BB}(x)$ under the most common Dirichlet boundary condition. Here, we consider the parametric regimes of this potential wherein the real part of $V(x)$ is zero or a well, these regimes are $\nu \in (1, 3] \cup [5, 7] \cup [9, 11]$. We show that the Dirichlet spectrum obtained by numerical integration of the Schrödinger equation and by the matrix diagonalization of $p^2 + V(x)$ are in agreement which eventually are the semi-classical WKB eigenvalues provided we use a concept of maximal turning points.

Scattering amplitudes for rationally extended PT symmetric complex potentials

Nisha Kumari[1], Rajesh Kumar Yadav[2], Avinash Khare[3], Bijan Bagchi[4], Bhabani Prasad Mandal[5]

Department of Physics, Banaras Hindu University, Varanasi-221005, India[1,5]*, Department of Physics, S. P. College, S. K. M. University, Dumka-814101, India*[2]*, Department of Physics, Savitribai Phule Pune University, Pune-411007, India*[3]*, Department of Physics, School of Natural Science, Shiv Nadar University, Greater Noida, UP-201314 India* [4]

nishaism0086@gmail.com [1], rajeshastrophysics@gmail.com[2], khare@physics.unipune.ac.in[3],bbagchi123@gmail.com[4], bhabani.mandal@gmail.com[5]

Scattering amplitudes for the rationally extended PT symmetric complex potentials In this paper, we consider the rational extensions of two different PT symmetric complex potentials namely the asymptotically vanishing Scarf II and asymptotically non-vanishing Rosen-Morse II [RM-II] potentials and obtain bound state eigenfunctions in terms of newly found exceptional X_m Jacobi polynomials and also some new type of orthogonal polynomials respectively. By considering the asymptotic behaviour of the exceptional polynomials, we obtain the reflection and transmission amplitudes for them and discuss the various novel properties of the corresponding amplitudes.

Time-evolution of quantum entropy associated with non-hermitian quantum dynamics

Hiroki Majima
Physics Department, General Education Division, Salesian Polytechnic
majima@salesio-sp.ac.jp

Non-hermitian Hamiltonians play an important role in the study of dissipative quantum systems. Traditionally non-hermitian Hamiltonians have been used to phenomenologically describe processes which lead to a non conservation on particle number, eg radioactive decay, scattering phenomena, and open quantum systems. In all such cases, the probability does not have to be conserved in general. Phase volume corresponds to entropy in classical theory, because the probability is represented by phase volume. Therefore, non-conservation of the probability causes changes of entropy

In this presentation, we consider the quantum entropy of systems described by non-hermitian Hamiltonians. The determinant of the density matrix in fact decays over time in non-hermitian Hamiltonian systems. From this, it can be presumed that the determinant of the density matrix in quantum systems corresponds to the phase volume in classical systems, from which the volume of a parallelepiped spanned by three vectors is the absolute value of the scalar triple product. According to this result, the determinant of the density matrix in quantum systems is comparable to phase volume compression in classical dissipative systems. We derive a relation between trace and determinant for the density matrix. The relation indicates that there is a difference relating to the degree of mixture between them. The phase volume is equivalent to entropy in classical theory. It implies that the determinant of the density matrix has relevance to entropy in quantum theory. We naturally show that quantum entropy is connected to the determinant of the density matrix in non-hermitian quantum systems which corresponds to phase volume compression in classical systems.

Next, we study an evolution equation with respect to a normalized density matrix in non-hermitian quantum systems. This evolution equation is derived by using a non-normalized density matrix. The generator of this evolution equation can be divided into two parts: The first part describes local unitary dynamics and the second term incoherent processes. In our previous work, we obtained the identity connect a change of the quantum entropy to a generator of the evolution equation. Applying the identity to the above evolution equation, we obtain a change in the quantum entropy attributed mainly to the incoherent part of the evolution equation for the density matrix. As a result, we can evaluate how the change in the quantum entropy is influenced from the incoherent part of the evolution equation in non-hermitian quantum systems.

$\mathcal{PT}$ symmetry for nonunitrary quantum walks and the extention to disordered systems

Ken Mochizuki, Hideaki Obuse
Department of Applied Physics, Hokkaido University
ken_mochizuki@eng.hokudai.ac.jp, hideaki.obuse@eng.hokudai.ac.jp

Recently, non-Hermitian systems with $\mathcal{PT}$ (combined parity and time-reversal) symmetry have been realized in various experiments, and many interesting phenomena have been observed in these systems. However, difficulty in controlling gain and loss effects makes most experimental systems have only a small number of elements. In contrast, quantum walks can overcome this difficulty.

Quantum walks recently attract attentions as a versatile platform for quantum computations and quantum simulations. Quantum walks describe quantum dynamics of particles (walkers) with several internal states by a time-evolution operator composed of coin operators, shift operators, and so on. The coin operator varies the internal states of walkers, and the shift operator changes the positions of walkers depending on the internal states. Quantum dynamics is described by acting the time-evolution operator to a quantum state at each time step. In 2012, a quantum walk built by optical fiber loops was experimentally implemented, in which highly tunable gain and loss effects are included with additional optical amplifiers [1]. Effects of gain and loss make the time-evolution operator nonunitary, which means the effective Hamiltonian is non-Hermitian. It is known that, in certain parameter regions, the system has entirely real quasienergy.

In the present work, we explicitly show that the reality of quasienergy obtained in Ref [1] stems from $\mathcal{PT}$ symmetry [2]. We find that parameters of operators must have correlations not only in spatial direction but also in time direction in order to preserve $\mathcal{PT}$ symmetry of the time-evolution operator. In addition, we numerically show that although parameters of the coin operator are random over the position space and the time-evolution operator does not hold $\mathcal{PT}$ symmetry in the explicit sense, the system can have entirely real quasienergy. Then, we discuss the reason why this occurs and study the effects of randomness on nonunitary quantum walks.

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A new type of PT-symmetric random matrix ensembles and PT-symmetric Quantum Chaos

Eva-Maria Graefe[1], **Steve Mudute-Ndumbe**[1], Sjoerd de Vries[1]
Department of Mathematics, Imperial College London[1]
e.m.graefe@imperial.ac.uk[1], steve.ndumbe10@imperial.ac.uk[1],
sjoerd.de-vries15@imperial.ac.uk[1]

We construct new ensembles of random matrices on the spaces of split-complex and split-quaternionic matrices, built using similar assumptions to the Gaussian Orthogonal, Unitary and Symplectic Ensembles, and compare some of the similarities and differences. We demonstrate that split-complex and split-quaternionic Hermitian matrices are isomorphic to PT-symmetric matrix Hamiltonians, and thus conjecture that these new ensembles can potentially be used as universality classes for PT-symmetric quantum systems whose closed equivalents are classically chaotic. We derive explicit results for the spectral densities of our new ensembles in the 2×2 case, and numerically identify properties in the more general $N \times N$ case. We further introduce PT-symmetric extensions of some paradigmatic models of quantum chaos and explore their spectral features.