Low-lying continuum states of drip-line Oxygen isotopes

Takaharu Otsuka
*University of Tokyo / MSU*

Koshiroh Tsukiyama, Rintaro Fujimoto
*University of Tokyo*
Motivation

Emission from doorway-state

A bound state is shifted to continuum suddenly through a nuclear reaction

V.S.

Emission from single-particle resonance
Eigenvalues of HO potential

Magic numbers
Mayer and Jensen (1949)

Spin-orbit splitting

Mayer & Jensen
Neutron single-particle energies at $N=20$ for $Z=8\sim20$

- **solid line**: full (central + tensor)
- **dashed line**: central only

$d_{3/2}$

- bound in F
- unbound in O

due to interplay between tensor and three-body forces

TO, et al. PRL 104, 012501 (2010)
One of the Backgrounds
Why is the drip line of Oxygen so near?

next issue → oxygen anomaly and continuum
The clue: Fujita-Miyazawa 3N mechanism (Δ-hole excitation)

Progress of Theoretical Physics, Vol. 17, No. 3, March 1957

Pion Theory of Three-Body Forces

Jun-ichi FUJITA and Hironari MIYAZAWA

Δ particle
m=1232 MeV
S=3/2, I=3/2

Miyazawa, 2007
Pauli blocking effect on the renormalization of single-particle energy

Renormalization of single particle energy due to $\Delta$-hole excitation

$\Rightarrow$ more binding (attractive)

Another valence particle in state $m'$

Pauli Forbidden

$\Rightarrow$ The effect is suppressed
Most important message with Fujita-Miyazawa 3NF

Renormalization of single particle energy

Effective monopole repulsive interaction

Pauli blocking

Monopole part of Fujita-Miyazawa 3-body force

same
(i) Δ-hole excitation in a conventional way

(a) G-matrix NN + 3N (Δ) forces

(b) \( V_{\text{low k}} \) NN + 3N (Δ, \( N^2 \) LO) forces

(ii) EFT with Δ

(iii) EFT incl. contact terms (\( N^2 \) LO)

Δ-hole dominant role in determining oxygen drip line

(c) 3-body interaction

(d) 3-body interaction with one more neutron added to (c)
Ground-state energies of oxygen isotopes

**NN force + 3N-induced NN force**

(Fujita-Miyazawa force)

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![Diagram showing energy levels of oxygen isotopes](image)
(i) Δ-hole excitation in a conventional way

- G-matrix $NN + 3N (\Delta)$ forces

(ii) EFT with $\Delta$

- $V_{\text{low } k} NN + 3N (\Delta, N^2 \text{LO})$ forces

(iii) EFT incl. contact terms ($N^2 \text{LO}$)

Δ-hole dominant role in determining oxygen drip line

$\Rightarrow$ to be discussed now
Continuum-coupled shell model (CCSM)

Hamiltonian: \[ H = H_0 + \hat{V} = \sum_j \tilde{\epsilon}_j n_j + \hat{V} \]

\[ H_0 = T + U_{WS} + V_{wall} = \sum_j \tilde{\epsilon}_j n_j \]

basis state-vector (denoted by \( j \)): bound states + discretized continuum states
wall very far (3000 fm, ~3000 basis states)

Hamiltonian is approximated by Gaussian.

V_{NN} +

\[ V \]

\[ d_{3/2} \]

\[ s_{1/2} \]
(i) $\Delta$-hole excitation in a conventional way

(a) G-matrix NN + 3N ($\Delta$) forces

(b) $V_{\text{low } k}$ NN + 3N ($\Delta$, $N^2$LO) forces

(ii) EFT with $\Delta$

$\Delta$-hole dominant role in determining oxygen drip line

phenomenological shell model
\[ \hat{V}(r) = \sum_{i=1,2} g_i (1 + a_i \sigma \cdot \sigma) e^{-r^2/d^2_i} \]

\[ d_{1,2} = 1.4, \ 0.7 \ \text{fm} \]

SDPF-M TBME = TBME of this \( V(r) \) for HO wave functions

\[ \langle 1s_{1/2} 0d_{3/2} | V | 1s_{1/2} 0d_{3/2} \rangle_{J=1,2} \]
\[ \langle 0d_{3/2} 0d_{3/2} | V | 0d_{3/2} 0d_{3/2} \rangle_{J=0,2} \]

under the assumption that 3-body force effect is included in SDPF-M interaction effectively

\( V(r) \) is fixed only by interaction
$^{24}\text{O} = ^{22}\text{O} + 2n$ in the space

**ground state**: $2n$ in $1s_{1/2}$

**excited states of $1^+$ and $2^+$**:

\[ |iJ^+\rangle = |1s_{1/2} \otimes id_{3/2}; J^+\rangle \]

**discretized continuum $id_{3/2}$ ($i = 1, 2, ...$)**

$1s_{1/2}$: solution of Woods-Saxon potential with observed $S_n$

**diagonalize $H$**

**Eigenfunctions**:

\[ |J_k^+\rangle = \sum_i c_i^{(J,k)} |iJ^+\rangle \]
Reaction mechanism

-> Doorway state
Removal of one proton and one neutron from $^{26}F$

$^9_{\text{Be}}(^{26}_{\text{F}},^{24}_{\text{O}})X$
C. Hoffman, M. Thoennessen et al.

knockout reaction @MSU (2009)
less probable
<= large $s_{1/2}$-$d_{3/2}$ neutron gap

continuum

Removal of one proton and one neutron from $^{26}F$

$^{26}F$

bound nucleus

$^{16}O$

-doorway state

$|1s_{1/2}0d_{3/2}; J_k^+\rangle$

$^{24}O$

excited states in $^{24}O$

$H^\text{CCSM}|J_k^+\rangle = E_k|J_k^+\rangle$

$^{16}O$

ground state

$1s_{1/2}$ is bound.
Kanungo et al. (2009)
Neutron single-particle energies at $N=20$ for $Z=8\sim20$

solid line: full (central + tensor)
dashed line: central only

$d_{3/2}$
bound in F
unbound in O
due to interplay between tensor and three-body forces

(a) neutron single-particle levels of $N=20$ isotones

88141620

energy (MeV)

TO, et al. PRL 104, 012501 (2010)
Removal of one proron and one neutron from $^{26}\text{F}$

Before the removal, neutron $d_{3/2}$ is well-bound in $^{26}\text{F}$ and can be described by a HO wave function.

Sudden removal $\rightarrow$ doorway state with HO $d_{3/2}$

Decay of neutron from this $d_{3/2}$ through overlap with continuum states:

$$\zeta_k^{(J)} \equiv \langle J^+_k|1s_{1/2}0d_{3/2}; J^+\rangle = \sum_i c_i^{(J,k)} \langle id_{3/2}|0d_{3/2}\rangle$$

$$p_k^{(J)} \equiv |\zeta_k^{(J)}|^2 \quad \rightarrow \quad \text{Spectrum of emitted neutron}$$
Low-lying Continuum Spectra in $^{24}$O

- **Doorway state $\rightarrow$ continuum states in $^{24}$O**
  \[ p^J_k = |\langle J^+_k | \Phi_{\text{doorway}} \rangle|^2 = |\sum_i C_i^{(k)} \langle id_{3/2} | 0d_{3/2} \rangle|^2 \]

- **Bound approximation:**
  Normal shell model with the same Hamiltonian: NO continuum effect

- **CCSM:** With continuum effect
  incl. residual interaction

- **no int.:** With continuum effect but no residual interaction.

- Continuum effect is about 1 MeV
- No bound excited state.
- $1^+-2^+$ splitting by 2-body interaction
- $1^+-2^+$ splitting is in good agreement with experiments.
Radial density (w.f.) of continuum states in $^{24}$O

- Notable difference between $1^+$ and $2^+$ states.
- The peak states in CCSM reproduce the behavior of “resonance wave” at far distance (phase shift of $\pi/2$).
Peak Energies of neutron emission

SPE as bound state

2 MeV

Lowering due to continuum effect

Continuum spectra are consistent with the shell evolution

Exp. : MSU (Hoffman et al), RIKEN (Elekes et al)
Convergence with respect to boundary condition

The results do not change so much if L is taken to be sufficiently large.

Even usual values of L~50 fm are not stable.
Comparison to

single-particle resonance
Effective phase shift and one-body reduction

Can many-body resonance be described by effective one-body problem?

CCSM: continuum spectra are obtained by taking the overlap between the doorway state and CCSM eigenstates in continuum.

Effective phase shift

We define effective phase shift by introducing 1-body reduction of CCSM wave function.

\[ |J_k^+\rangle = \sum_i c_i^{(J,k)} |1s_{1/2} \otimes id_{3/2}; J\rangle \]

\[ =: |1s_{1/2} \otimes \tilde{d}_{3/2}; J, k; J\rangle \]

\[ |\tilde{d}_{3/2}; J, k; J\rangle = \sum_i c_i^{(J,k)} |id_{3/2}\rangle \]

One can then obtain phase shift, and can use it for calculating the cross section.

\[ \sigma_J = \frac{4\pi}{k^2} (2l + 1) \sin^2 \delta_J \]
Effective phase shift and one-body reduction

- CCSM (doorway state approach) and effective phase shift approach give very similar results for peak positions.
- Notable difference appears for the width of $2^+$ in $^{24}$O.
  - Doorway state decays faster.

<table>
<thead>
<tr>
<th>Unit : MeV</th>
<th>$^{23}$O</th>
<th>$^{24}$O</th>
</tr>
</thead>
<tbody>
<tr>
<td>states</td>
<td>$3/2^+$</td>
<td>$1^+$</td>
</tr>
<tr>
<td>CCSM $E$</td>
<td>0.92</td>
<td>1.35</td>
</tr>
<tr>
<td>CCSM $\Gamma$</td>
<td>0.11</td>
<td>0.28</td>
</tr>
<tr>
<td>Phase shift $E$</td>
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</table>

50% longer life time
Phase shift

$\Delta ^{23}\text{O} \ 3/2^+$

$\tan \delta_J$

$\tan \delta_J$

$E \ [\text{MeV}]$

$^{24}\text{O} \ 1^+$

$^{24}\text{O} \ 2^+$
Although resonance state and doorway state are different, continuum spectra are similar.

What is the meaning of single-particle resonance states in complex dynamical processes such as multi-nucleon transfer heavy-ion reactions???

Time scale of the heavy-ion reaction may be shorter than the resonance life time.

Coupling to continuum lowers the (peak) energies by more than 1 MeV for oxygen isotopes.