Mini-symposium on non-Hermitian quantum mechanics
2022/07/04 @ U Tokyo (Hongo campus)

筑波大学
University of Tsukuba

Extreme Universe
A New Paradigm for Spacetime and Matter from Quantum Information
Grant-in-Aid for Transformative Research Areas (A)

Correlation effects on Z classification of non-Hermitian topology

by Tsuneya Yoshida (U. Tsukuba)

with Yasuhiro Hatsugai (U. Tsukuba)

- TY-Hatsugai, PRB **104**, 075106 (2021)
- TY-Hatsugai, arXiv:2205.09333

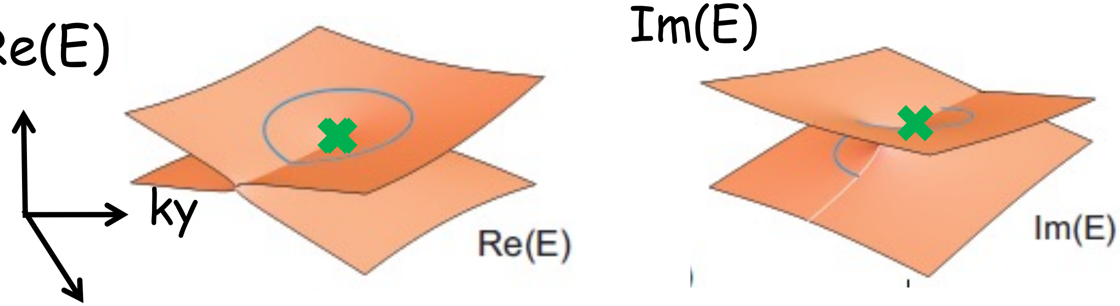
Introduction

Novel topological phenomena in non-Hermitian systems

Exceptional point

non-Hermitian topology:
band touching for both Re(E) and Im(E)

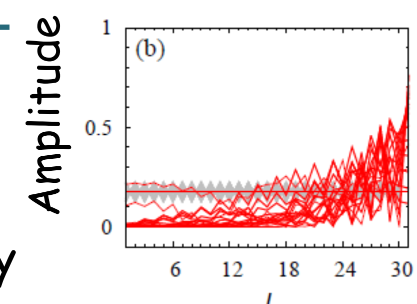
Re(E) Im(E)



H. Shen, B. Zhen, and L. Fu PRL (2018)

Non-Hermitian skin effects

Non-Hermitian topology
Most of eigenstates are localized at the boundary



Sites

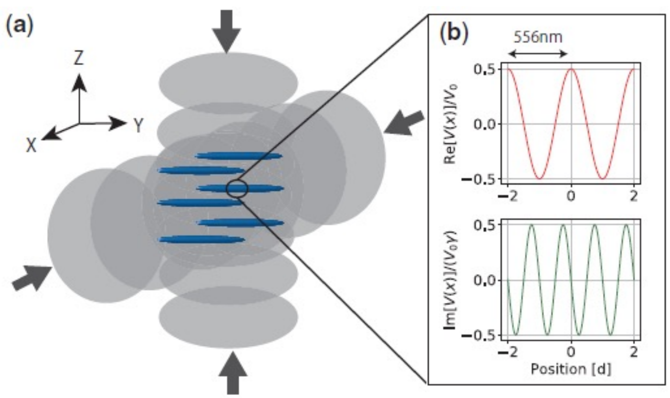
S. Yao and Z. Wang PRL (2018)

Non-interacting case
(band theory)

Unique phenomena in non-Hermitian systems

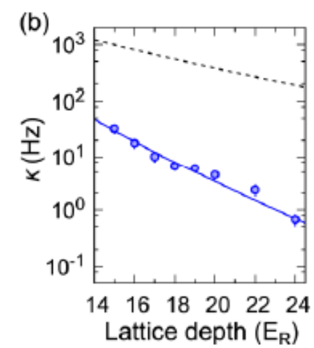
Dissipation in cold atoms

One-particle loss



Y. Takasu et al., PTEP(2020)

Two-particle loss



T. Tomita et al, PRA(2019)

Interaction can be tuned in experiments

Cold atoms:

platforms of strong interactions vs. non-Hermitian topology

～nH topology vs. strong correlations～

At the non-interacting level, classification of nH topology is done

AZ class	Gap	Classifying space	$d=0$	$d=1$	$d=2$	$d=3$
AI	P	$\mathcal{R}_1$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
	L_r	$\mathcal{R}_0$	$\mathbb{Z}$	0	0	0
	L_i	$\mathcal{R}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
BDI	P	$\mathcal{R}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
	L_r	$\mathcal{R}_1$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
	L_i	$\mathcal{R}_2 \times \mathcal{R}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z} \oplus \mathbb{Z}$	0
D	P	$\mathcal{R}_3$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
	L	$\mathcal{R}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
DIII	P	$\mathcal{R}_4$	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
	L_r	$\mathcal{R}_3$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
	L_i	$\mathcal{C}_0$	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AII	P	$\mathcal{R}_5$	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
	L_r	$\mathcal{R}_4$	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
	L_i	$\mathcal{R}_6$	0	0	$2\mathbb{Z}$	0

Z. Gong, et al., PRX (2018)
K. Kawabata et al., PRX (2019)
H. Zhou and J.Y Lee PRB (2019)

Fate of point-gap topology under correlations?

$$H = H^\dagger$$

Reduction of Z classification:

(1D, with chiral symm.)

Free fermions		Correlations
$\mathbb{Z}$	→	$\mathbb{Z}_4$

Fidkowski-Kitaev PRB(2010)

Contents

～Fate of point-gap topology under strong correlations～

Part1: 0D with chiral symmetry: $Z \to Z_2$

TY-Hatsugai, PRB **104**, 075106 (2021)

Part2: 1D with spin-parity symmetry: $Z \times Z \to Z$ (synthetic/spatial dim.)

TY-Hatsugai, arXiv:2205.09333

cf.) line-gap topo.: Xi et al, Sci. Bull. (2021)

Warm up (Hermitian):

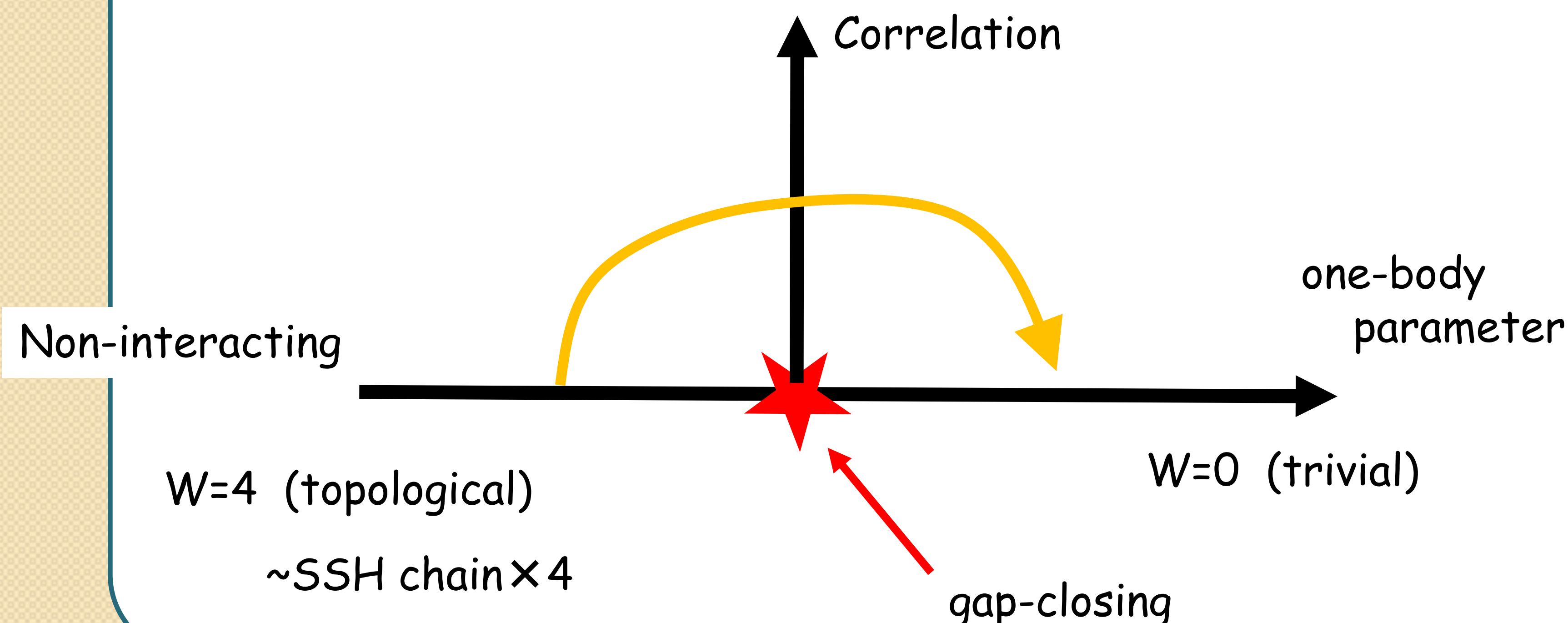
$H = H^\dagger$

Reduction of Z classification:

(1D, with chiral symm.)

Free fermions		Correlations
$\mathbb{Z}$	→	$\mathbb{Z}_4$

Fidkowski-Kitaev PRB(2010)



Part 1

~0-dimensional system with chiral symmetry~

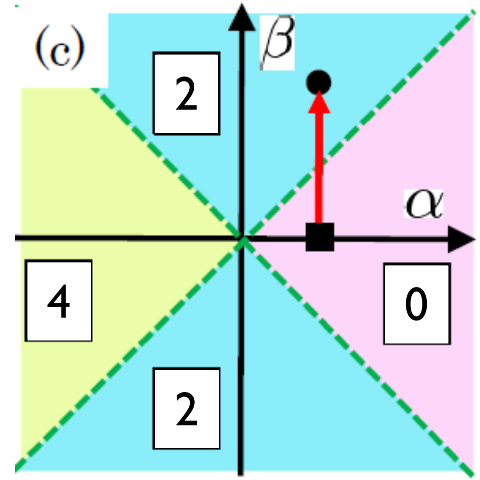
TY-Hatsugai, PRB **104**, 075106 (2021)

Overview

0D, chiral symmetry

Non-interacting case

nH point-gap topology is characterized by a Z-invariant

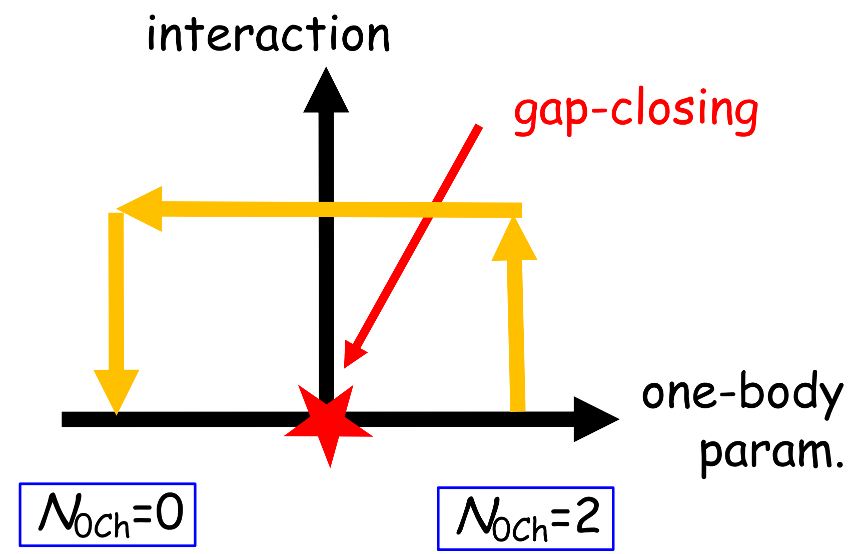


Correlated case

Topological states with

$N_{0Ch}=0$ and $N_{0Ch}=2$

are connected without gap closing



Free fermions $\mathbb{Z}$ → Correlations $\mathbb{Z}_2$

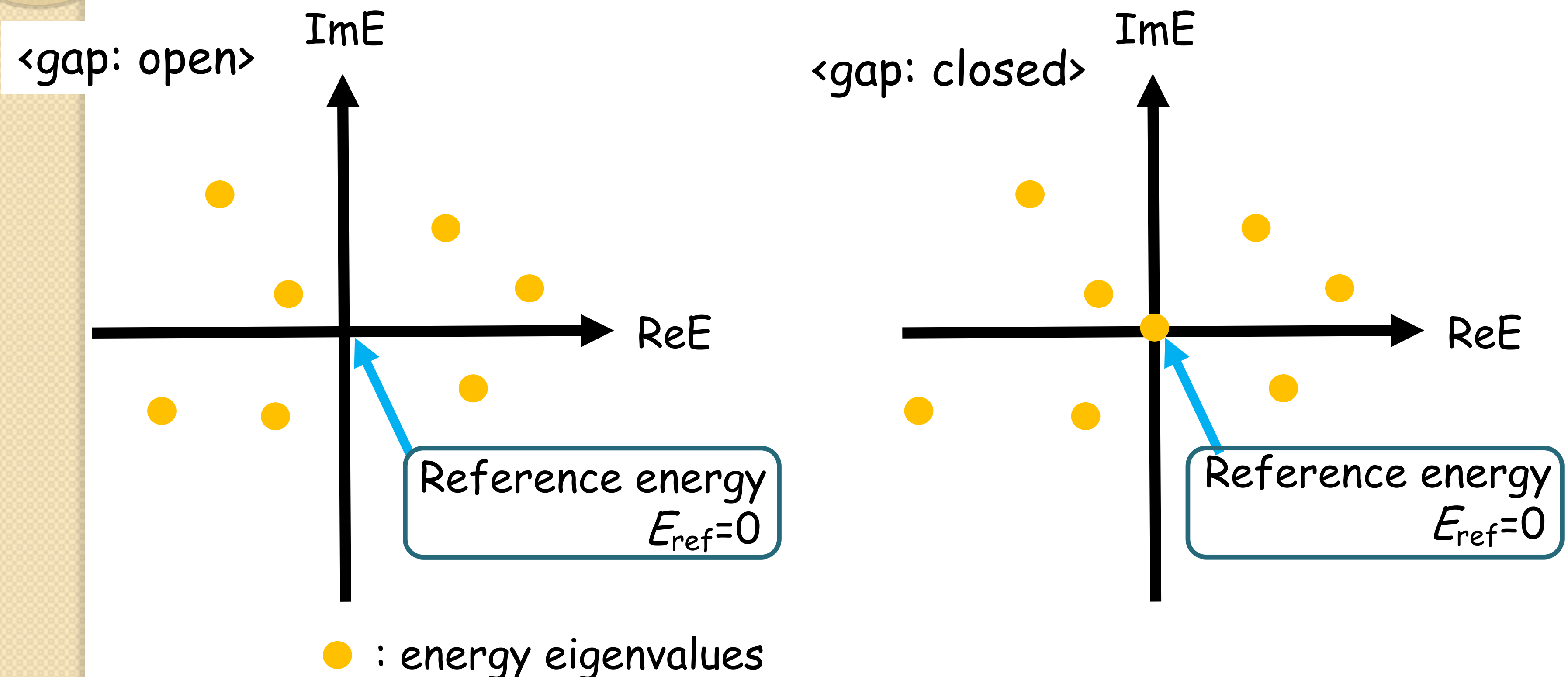
TY-Hatsugai, PRB **104**, 075106 (2021)

Gong et al., PRX (2018)

Kawabata et al., PRX (2019)

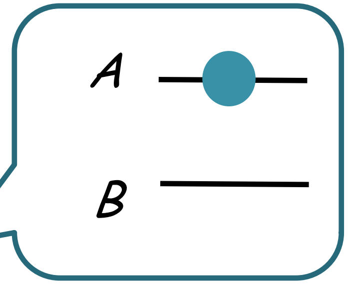
Definition of point-gap

$$\Delta_{\rm pt} = \min|E_n - E_{\rm ref}|$$



Hamiltonian (one-body)

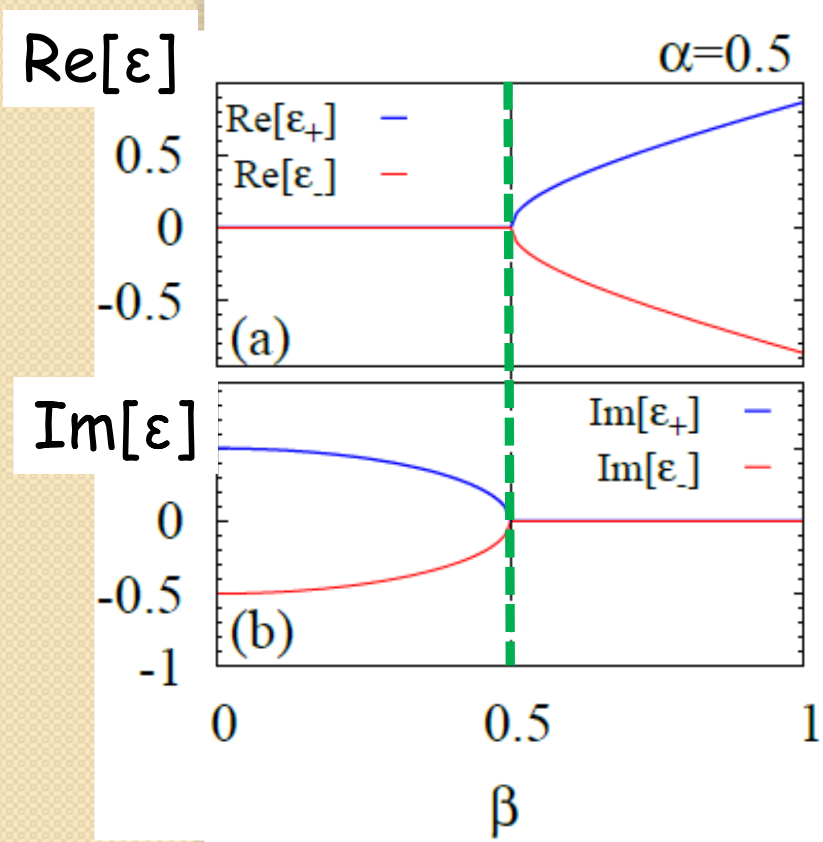
$$h = \begin{pmatrix} i\alpha & \beta \\ \beta & -i\alpha \end{pmatrix} \begin{matrix} A \\ B \end{matrix}$$



spinless fermion

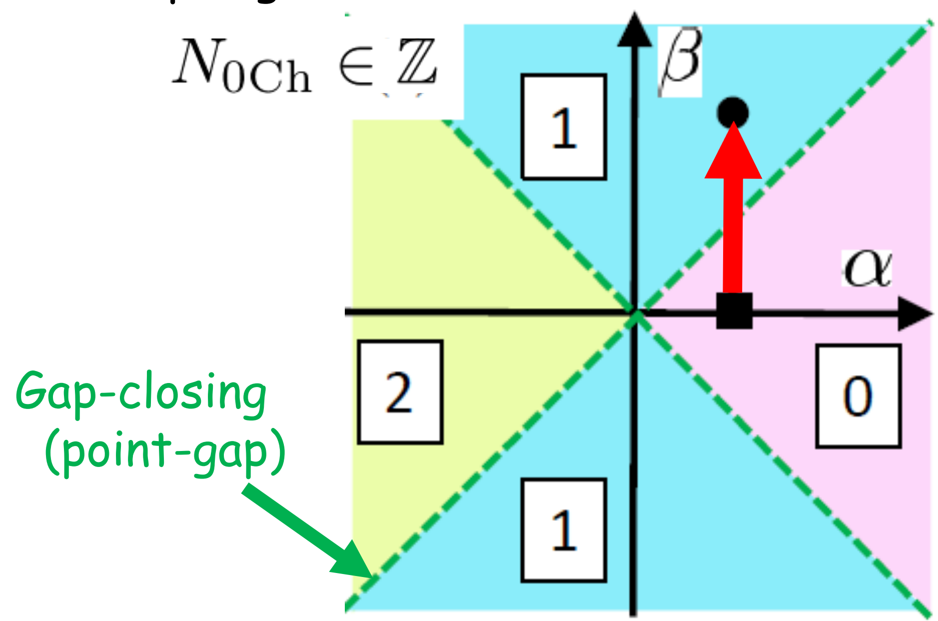
$$\alpha, \beta \in \mathbb{R}$$

$$\tau_3 h^\dagger \tau_3 = -h$$



<topological invariant>

$$N_{0\mathrm{Ch}} \in \mathbb{Z}$$



Many-body Hamiltonian

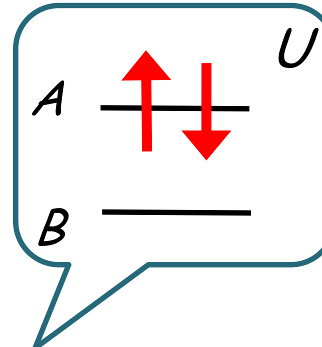
fermion with spin 1/2

$$\hat{H} = \hat{H}_0 + \hat{H}_U$$

r=1/2

$$\hat{H}_0 = \hat{\Psi}^\dagger \begin{pmatrix} h & 0 \\ 0 & rh \end{pmatrix} \hat{\Psi}$$

$$\hat{\Psi} = (\hat{c}_{A\uparrow}, \hat{c}_{B\uparrow}, \hat{c}_{A\downarrow}, \hat{c}_{B\downarrow})^T$$



$$\hat{H}_U = U \sum_{l=A,B} \left(\hat{n}_{l\uparrow} - \frac{1}{2}\right)\left(\hat{n}_{l\downarrow} - \frac{1}{2}\right)$$

$$U \geq 0$$

Chiral symmetry (many-body)

$$\hat{\Xi}\hat{H}\hat{\Xi}^{-1} = \hat{H},$$

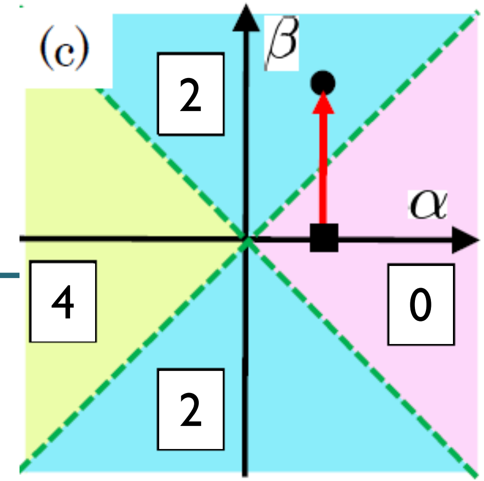
$$\hat{\Xi} = \prod_s (\hat{c}^\dagger_{As} + \hat{c}_{As})(\hat{c}^\dagger_{Bs} - \hat{c}_{Bs})\mathcal{K},$$

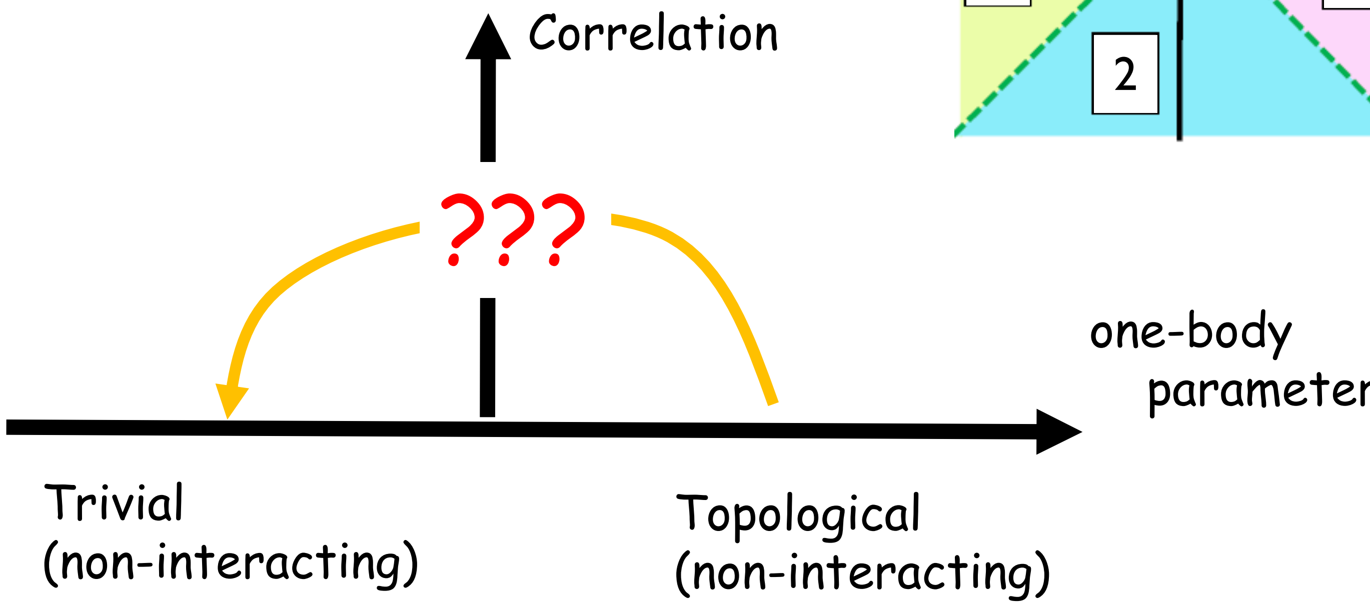
If U=0,

$$\tau_3 h^\dagger \tau_3 = -h$$

Hatsugai, JPSJ (2006)
Gurarie, PRB (2011)

Non-Hermitian case:



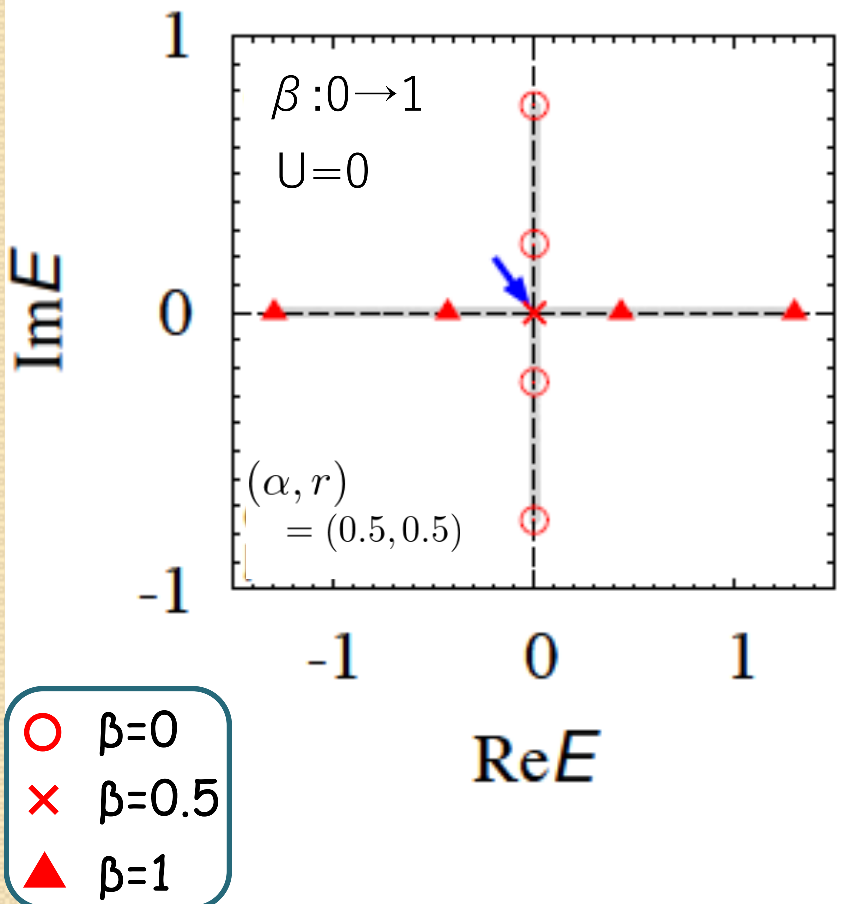


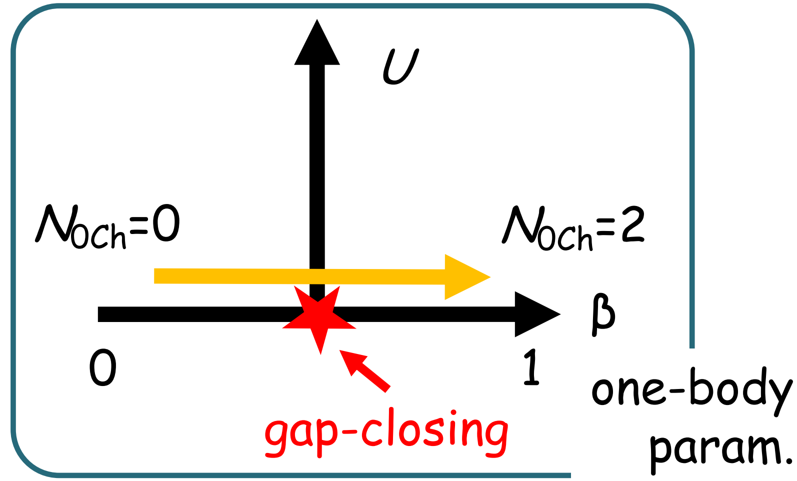
TY-Hatsugai, PRB **104**, 075106 (2021)

Spectral flow β :0→1

@U=0

$$(N_{\rm tot}, 2S^z_{\rm tot}) = (2, 0)$$

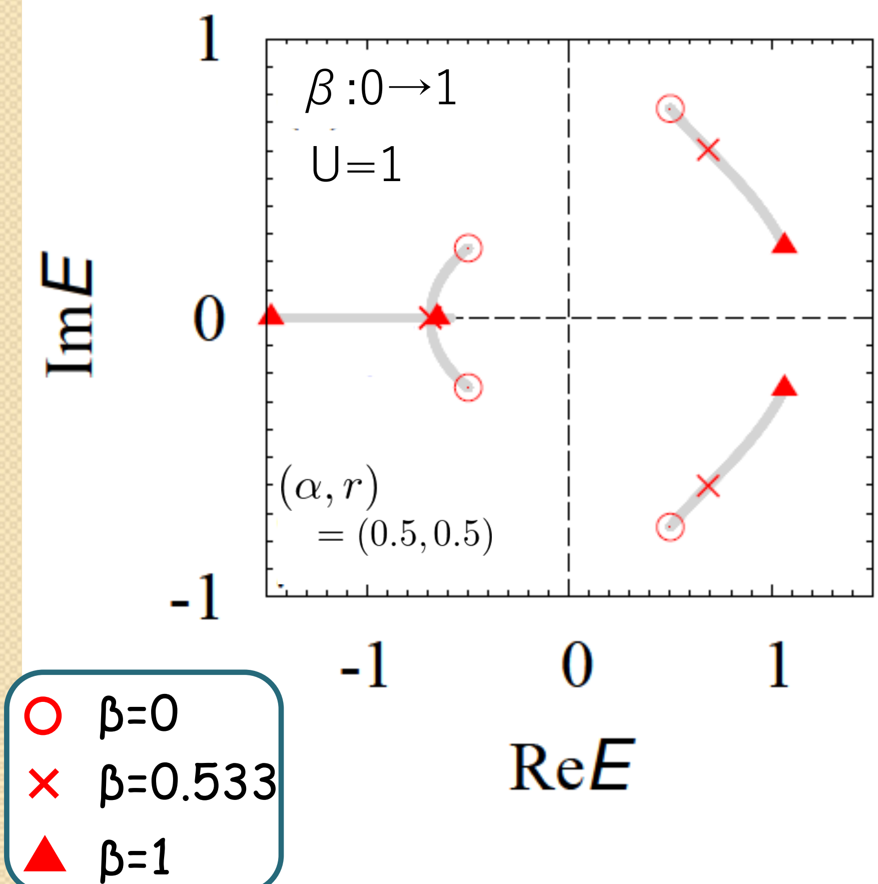


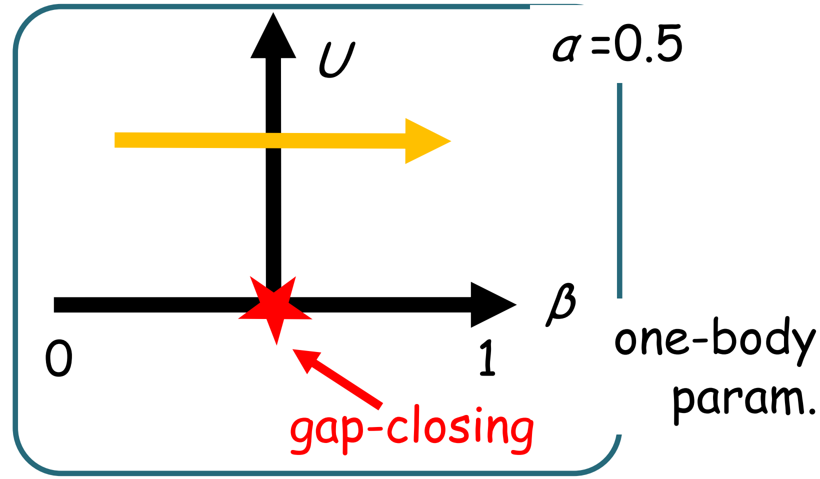


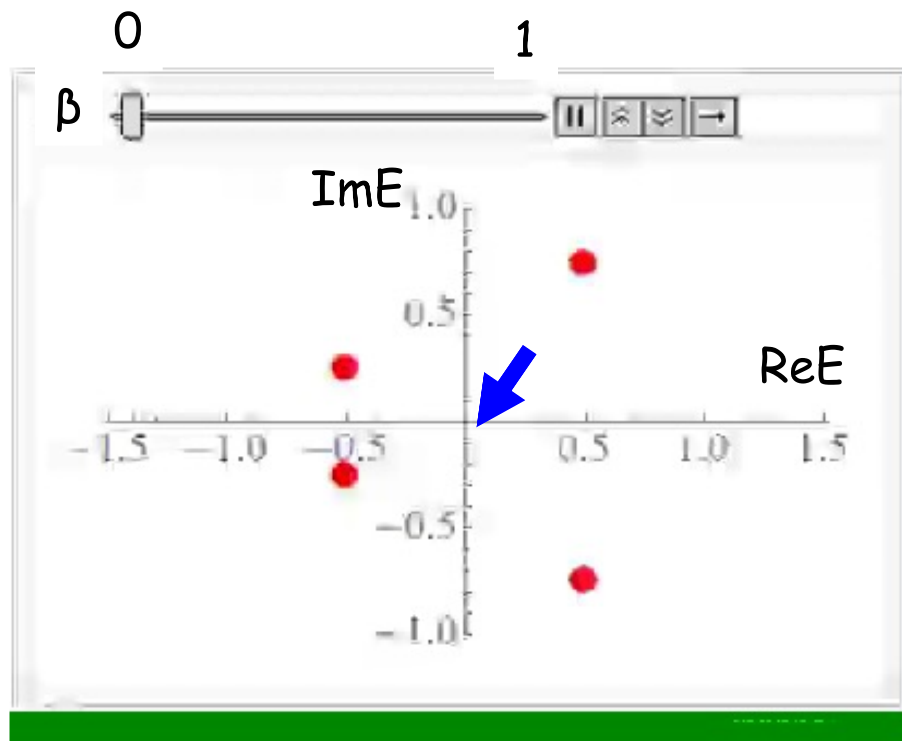
Spectral flow β :0→1

@U=1

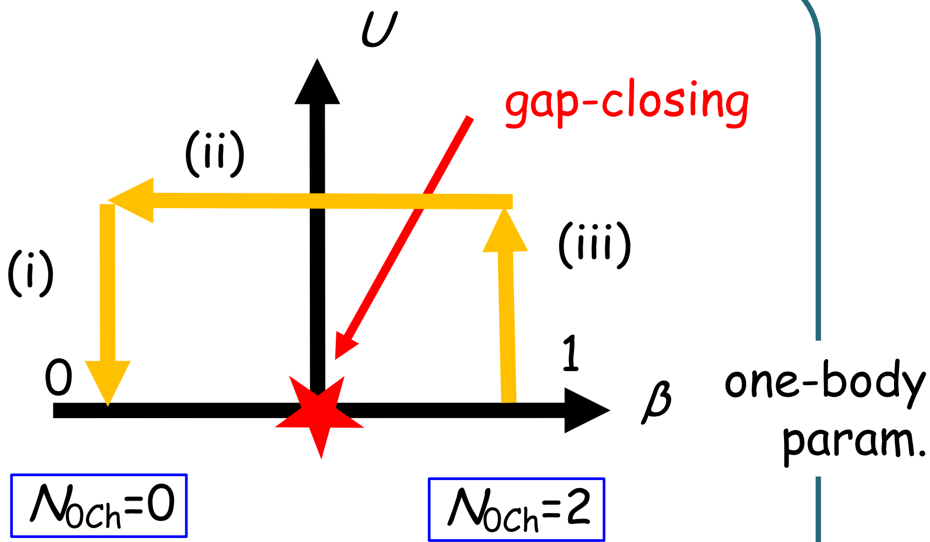
$$(N_{\rm tot}, 2S^z_{\rm tot}) = (2,0)$$





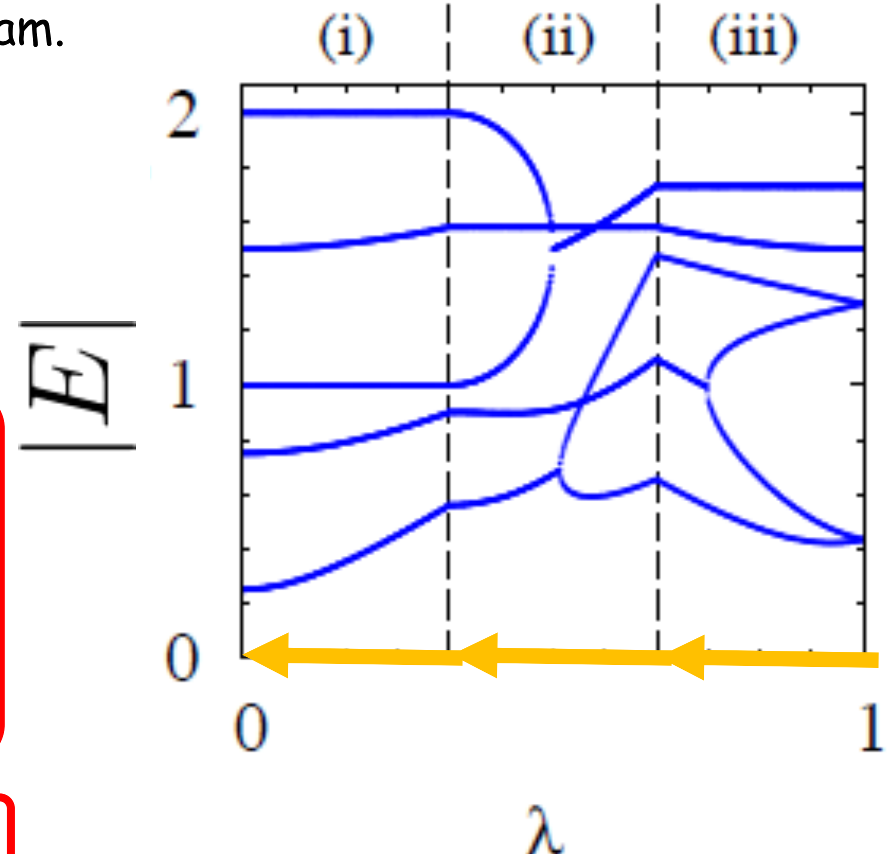


No gap-closing@U=1



cf.) gap-closing:

$$|E| = 0$$



In the presence of interactions,

$N_{0\mathrm{ch}}=0$ and $N_{0\mathrm{ch}}=2$

can be smoothly connected

without gap-closing !

→ $Z \rightarrow Z_2$

Perspective from topo. invariants

free → correlated

$\mathbb{Z} \rightarrow \mathbb{Z}_2$

Non-interacting

- H is quadratic Hamiltonian

$$\hat{H}_0 = \hat{\Psi}^\dagger \begin{pmatrix} h & 0 \\ 0 & rh \end{pmatrix} \hat{\Psi}$$

chiral symm. of one-body Hami.

$$\tau_3 h^\dagger \tau_3 = -h$$

Z-invariant

# of negative eigenvalues of $-i\tau_3 h$

Correlated

Many-body chiral symm.

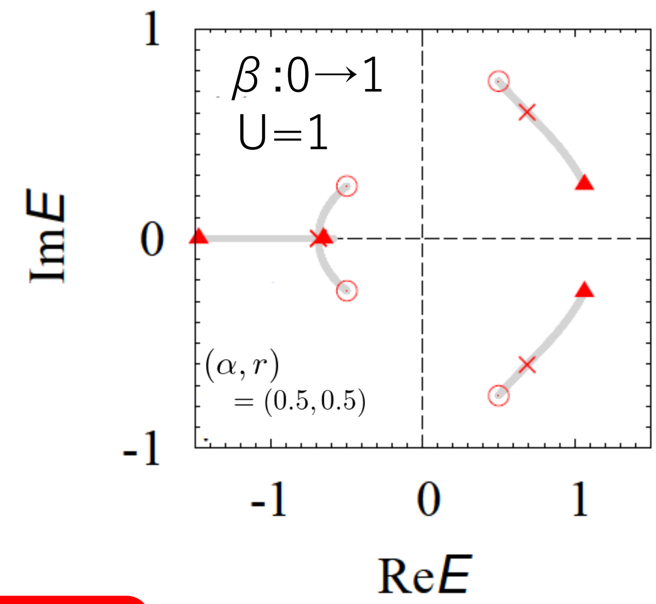
$$\hat{\Xi}\hat{H}\hat{\Xi}^{-1} = \hat{H},$$

$$\hat{\Xi} = \hat{U}K$$

Z_2-invariant

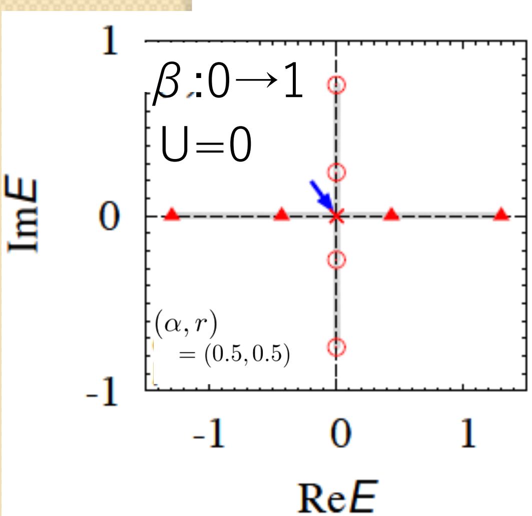
$$\nu = \mathrm{sgn}[\det \hat{H}]$$

Summary of part 1



Point-gap topology for 0D AIII

$Z \rightarrow Z_2$



TY-Hatsugai, PRB **104**, 075106 (2021)

Part 2

~1-dimensional system with spin-parity symmetry~

TY-Hatsugai, arXiv:2205.09333

Overview

~ 1D topology with spin-parity and charge U(1) symmetry ~

Synthetic dimension

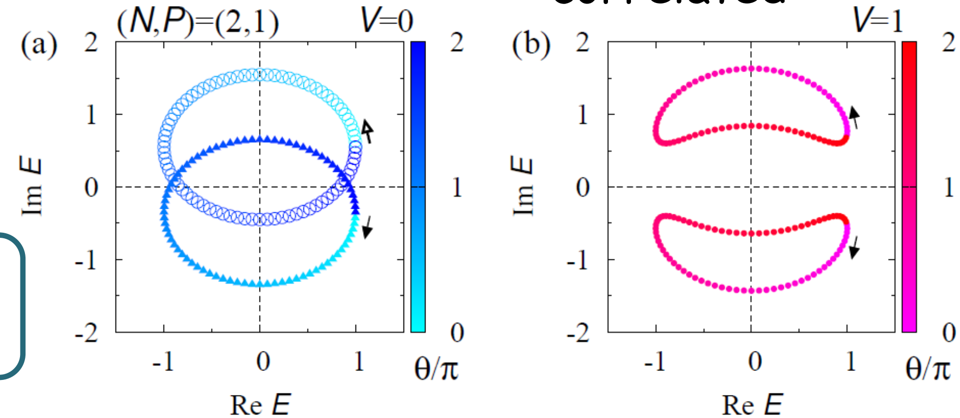
free		correlated
ZxZ	→	Z

constraint on spin-flipping terms (spin-parity)

one-body: forbidden

two-body: allowed

free correlated

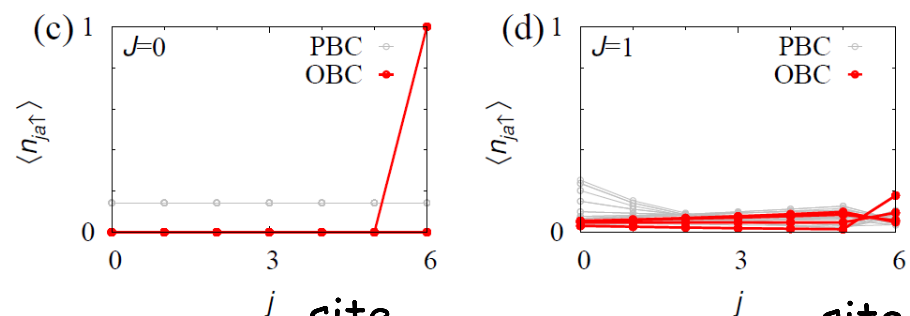


Spatial dimension

Reduction: ZxZ→Z

Destruction of
a skin effect by correlations

free correlated



site site

TY-Hatsugai, arXiv:2205.09333

Thank you!