Non-Hermitian exceptional boundary mode and its application to topological laser

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Collaborators: Yuto Ashida, Takahiro Sagawa

KS, Y. Ashida, and T. Sagawa, Nat. Commun. **11,** 5745 (2020).
KS, Y. Ashida, and T. Sagawa, Phys. Rev. B **105,** 235426 (2022).

Outline

Introduction

- Non-Hermitian topology

Exceptional edge mode

- Fundamental concept
- Numerical results on a toy model

Applications to topological lasers

- Toy models and numerical results
- Proposal of photonic realization

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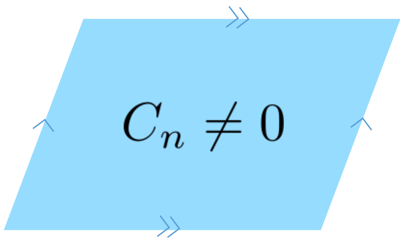
Band Topology and Bulk-Edge Correspondence

Bulk (periodic boundary)

$$H(\mathbf{k})|\psi_n(\mathbf{k})\rangle = E_n(\mathbf{k})|\psi_n(\mathbf{k})\rangle$$

(Eigenequation of each wavenumber sector)

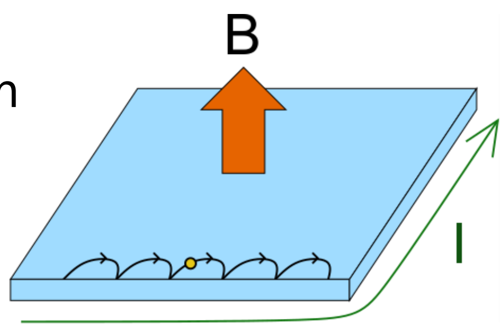


Chern number

$$C_n = \frac{1}{2\pi}\int_{\mathrm{BZ}} \nabla_{\mathbf{k}} \times \mathbf{A}_n(\mathbf{k}) \cdot d\mathbf{S}$$

$$\mathbf{A}_n(\mathbf{k}) = i\langle\psi_n(\mathbf{k})|\nabla_{\mathbf{k}}|\psi_n(\mathbf{k})\rangle$$

e.g. Quantum Hall effect

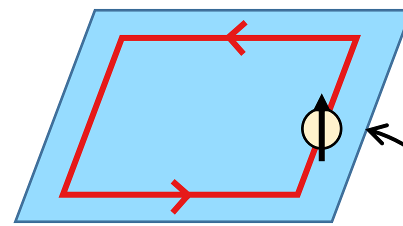


Bulk-edge correspondence

$\Leftrightarrow$

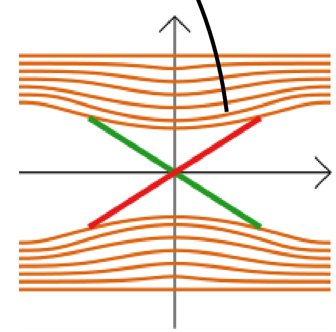
Edge (open boundary)

$$H|\Psi_n\rangle = E_n|\Psi_n\rangle$$



Edge modes

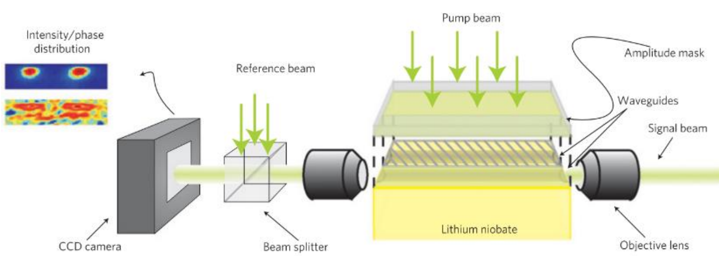
- **Localized** states
- Gapless dispersions
- **Backscattering-free** (unidirectional) current
- **Robustness** against disorders



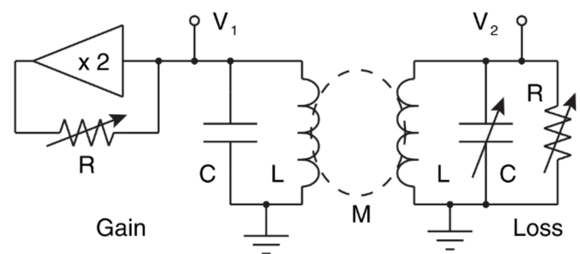
Topology of Non-Hermitian Hamiltonians

Non-Hermitian $H \neq H^\dagger$

- Description for systems **with gain-loss** and/or **nonreciprocity**
  e.g. photonics, electrical circuits, cold atoms



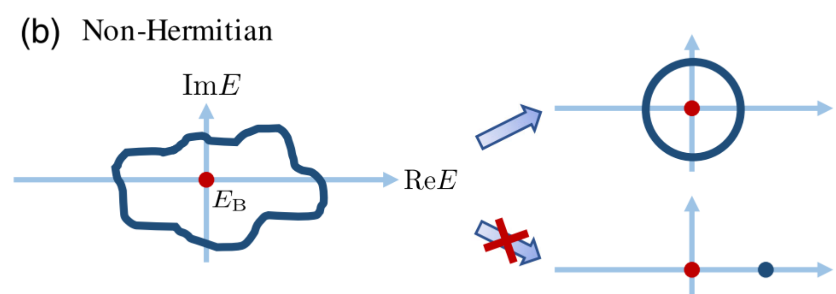
C. E. Rüter *et al.* Nat. Phys. 6, 192 (2010)



J. Schindler *et al.* Phys. Rev. A 84, 040101 (2011)

Unique topological features

- Novel classification utilizing complex eigenvalues
- Topological protection of **exceptional points**

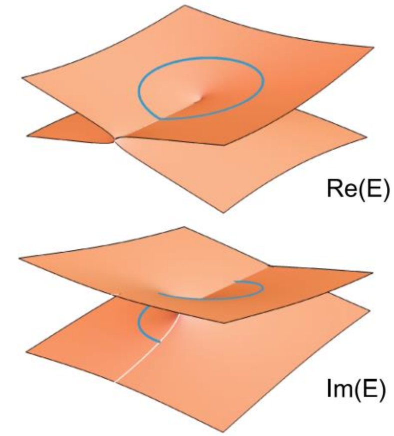


Z. Gong *et al.* PRX 8, 3 (2018)

Exceptional Points: Non-Hermitian Topological Gapless Points

Exceptional points (EPs)
= Nondiagonalizable gapless points
(cf. Jordan normal form)

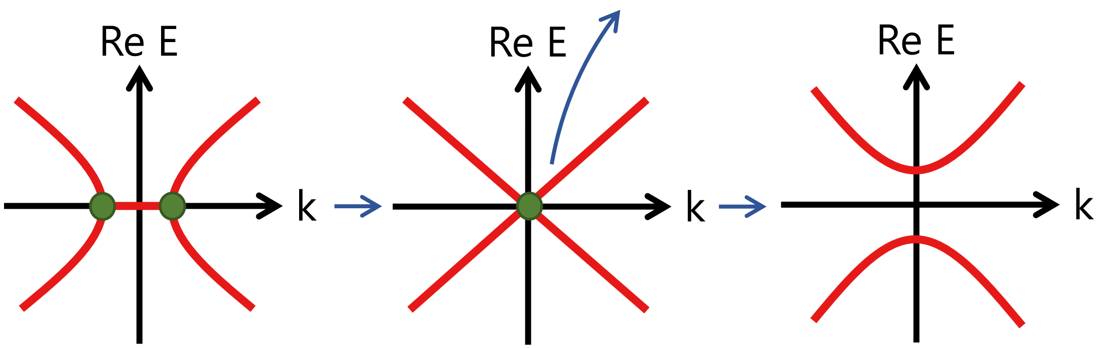
cf) $H(k) = \begin{pmatrix} k & 1 \\ 0 & -k \end{pmatrix}$ exceptional point at $k = 0$



H. Shen *et al.* PRL 120, 146402 (2018)

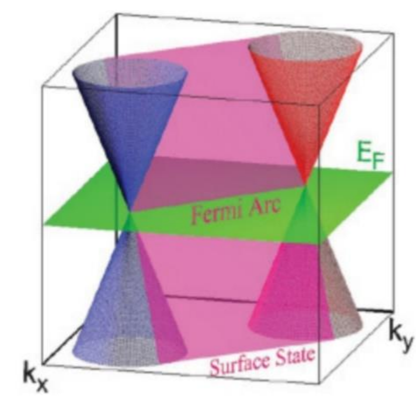
Topologically protected!!

→ Removed only via **pair annihilations**



Green points: exceptional points

Analogous to Weyl points



X. Wan *et al.*, PRB 83, 205101 (2011).

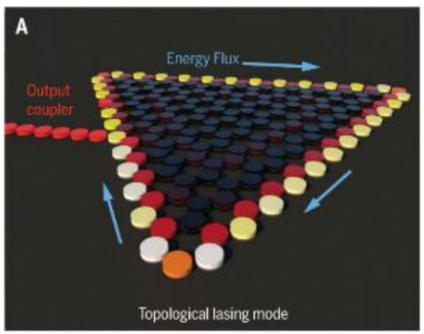
Topological Laser: Application of Non-Hermitian Topology

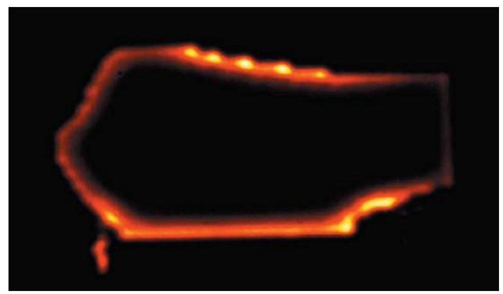
Topological laser

⇒ Amplifying boundary modes

Advantage: robustness against disorders

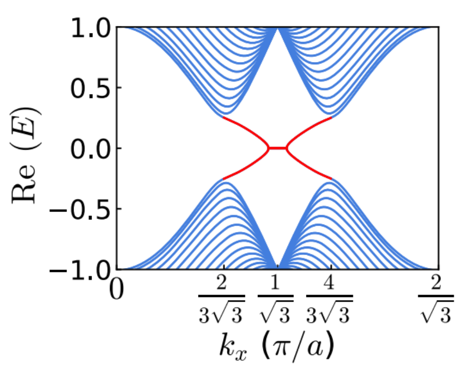
- Topological insulators + **judicious gain** at the edge of the sample

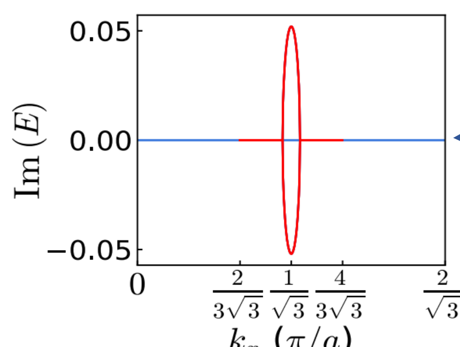




- B. Bahari *et al.* Science 358, 636 (2017).
- G. Harari *et al.* Science 359, 1230 (2018).
- M. A. Bandres *et al.* Science 359, 1231 (2018).

- **Symmetry** protected lasing mode





PT symmetry

- K. Kawabata, K. Shiozaki, M. Ueda, M. Sato PRX 9, 041015 (2019).
- A. Y. Song *et al.* PRL 125, 033603 (2020).

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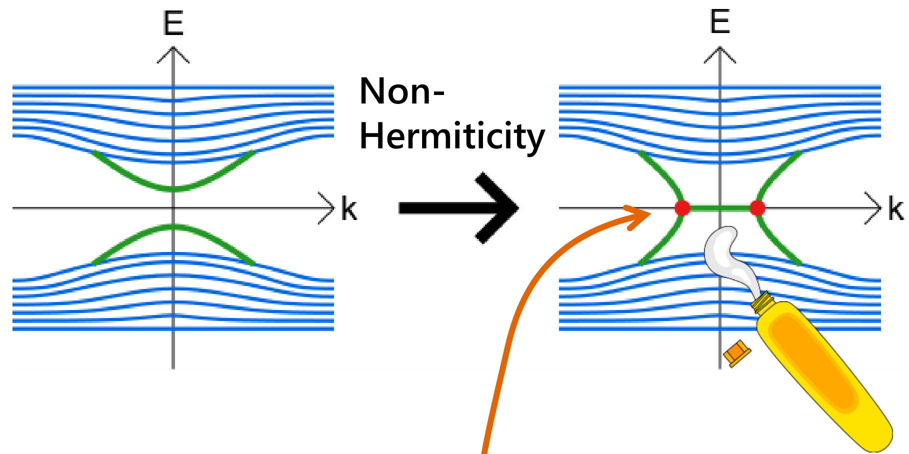
Exceptional Edge Modes Independent of Bulk Topology

Exceptional edge modes
(unique to non-Hermitian systems)

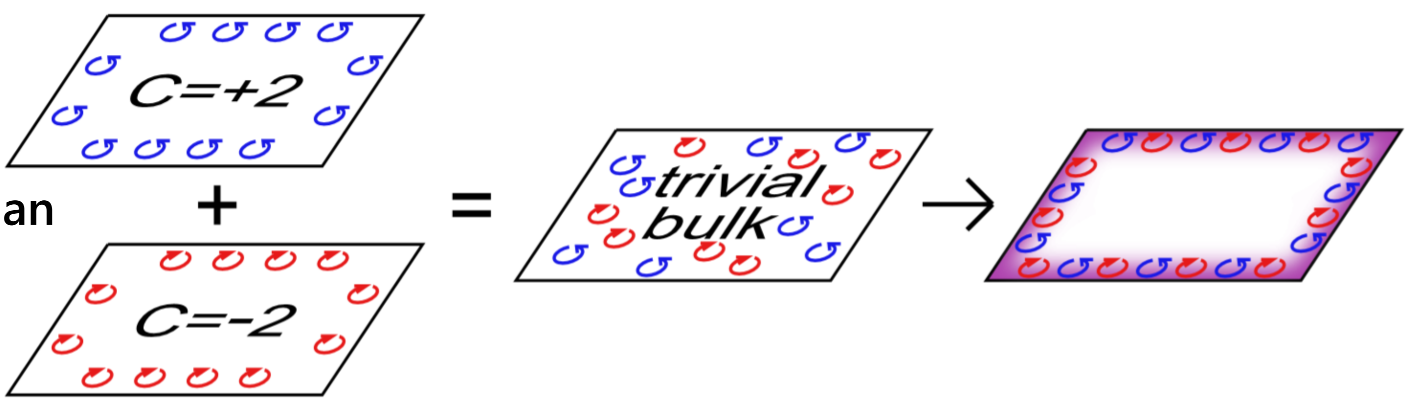


- Independent of the bulk topology
  → **Breakdown of the bulk-edge correspondence**
  (different from skin effect)

Exceptional points protect edge modes like "glue"

Typical construction procedure

Non-Hermitian coupling



Toy Model for Exceptional Edge Modes

Toy model (tight-binding)

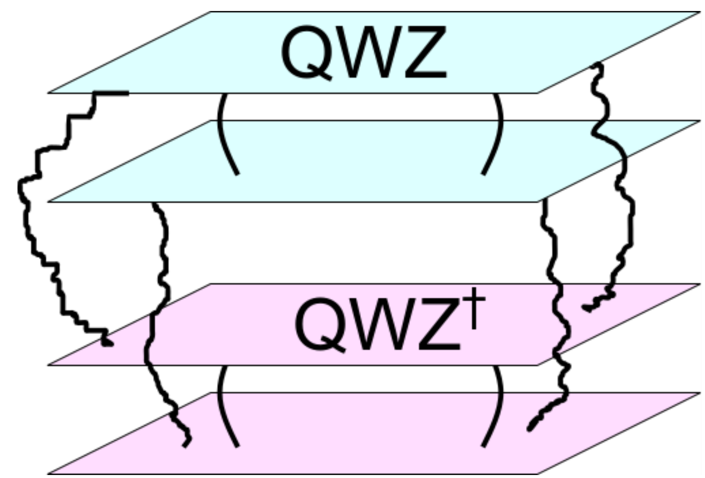
$$H = \begin{pmatrix} H_{\mathrm{QWZ}} & cI & ia\beta\sigma_x & -i\alpha\sigma_x \\ cI & H_{\mathrm{QWZ}} & i\alpha\sigma_x & ia\beta'\sigma_x \\ ia\beta\sigma_x & -i\alpha\sigma_x & H^*_{\mathrm{QWZ}} & cI \\ i\alpha\sigma_x & ia\beta'\sigma_x & cI & H^*_{\mathrm{QWZ}} \end{pmatrix}$$

Chern insulator

Non-Hermitian coupling

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$H_{\mathrm{QWZ}} =$ 2 × 2 Hamiltonian of the QWZ model (Chern insulator)

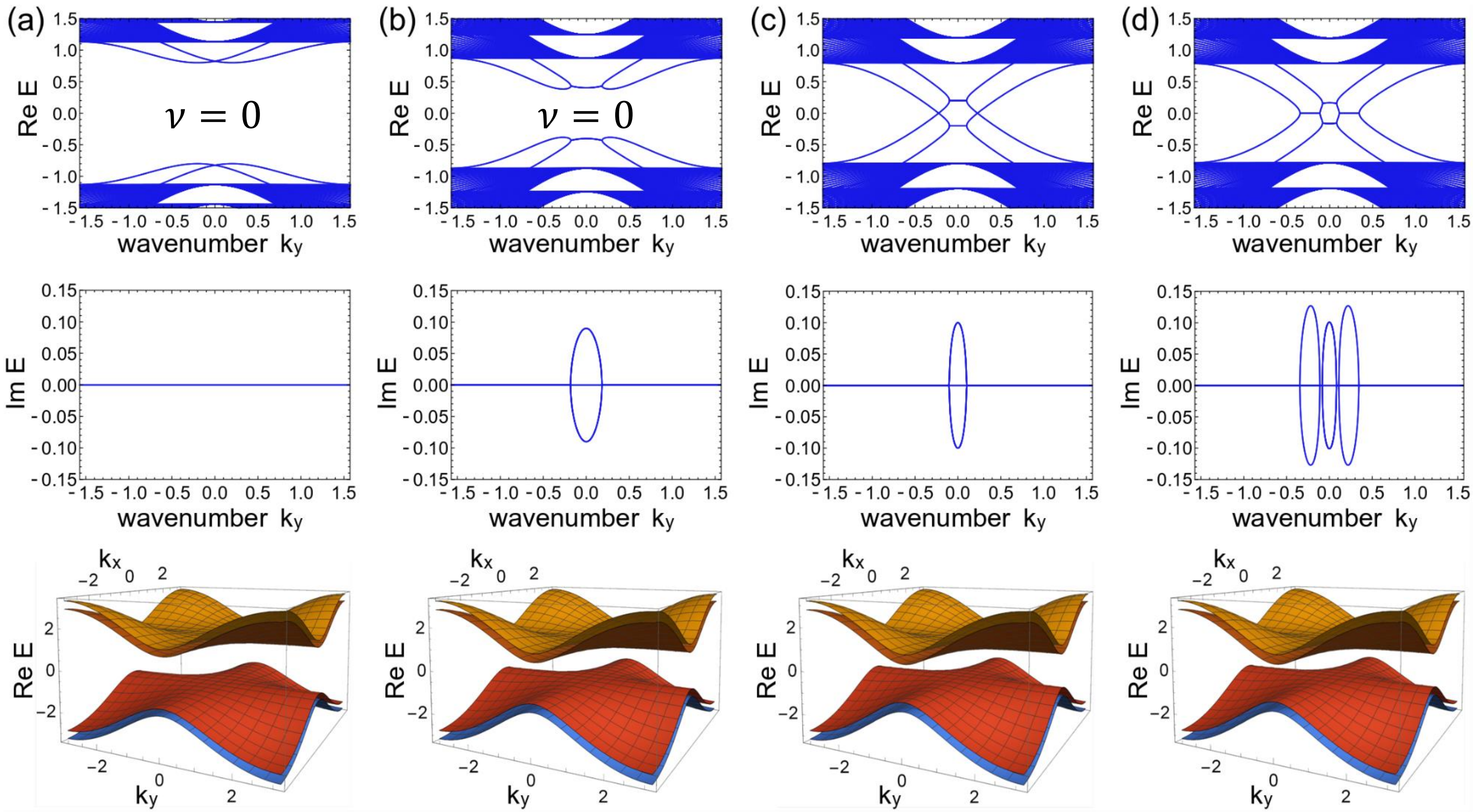


Curve: Hermitian coupling
Wave curve: non-Hermitian coupling

Ref. QWZ model: X. L. Qi, Y. S. Wu, & S. C. Zhang PRB 74, 085308 (2006).
BHZ model: B. A. Bernevig, T. L. Hughes, & S. C. Zhang Science 314, 1757 (2006).

Independence of the Bulk Topology

0 Non-Hermiticity large



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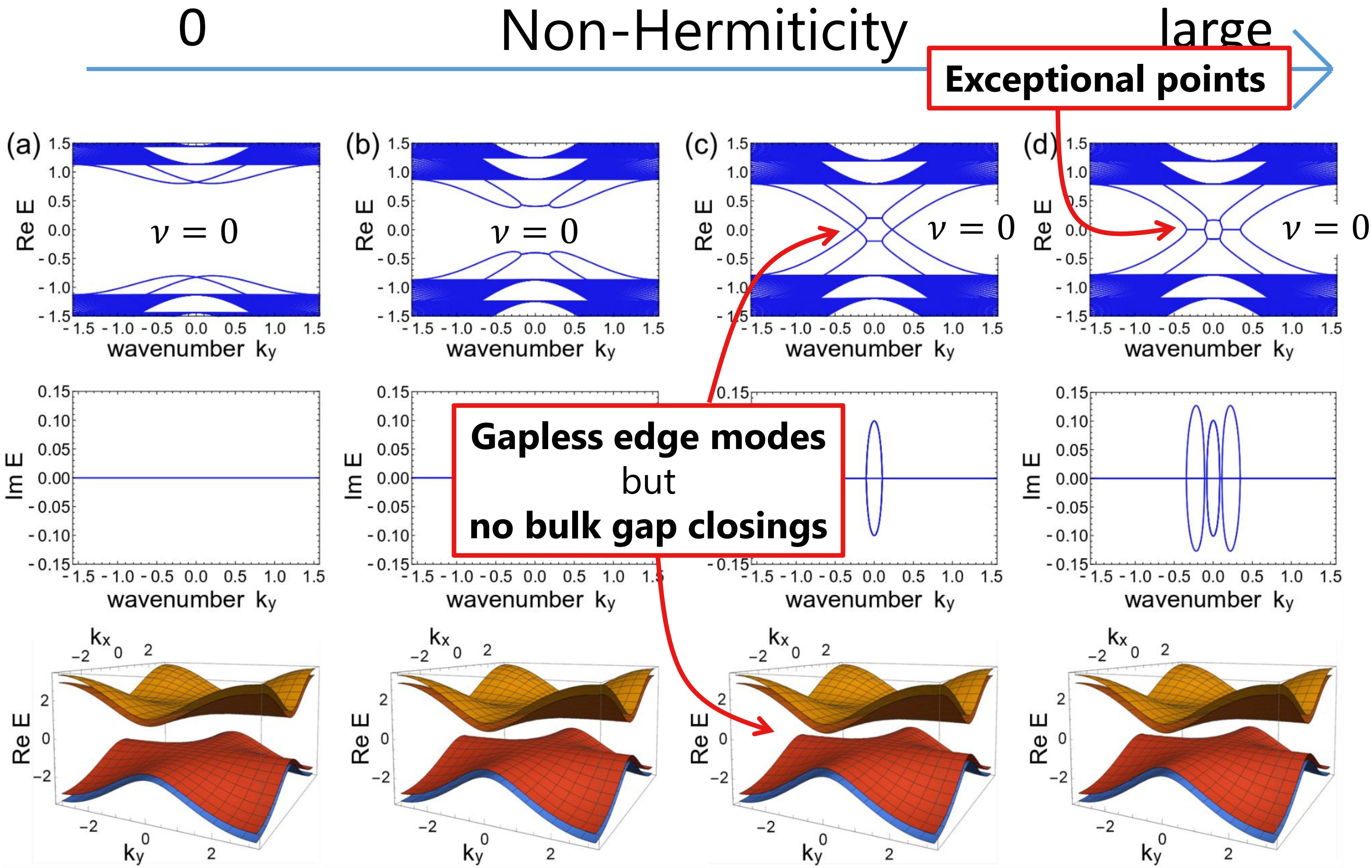


Topological Protection of Exceptional Points Under Symmetries

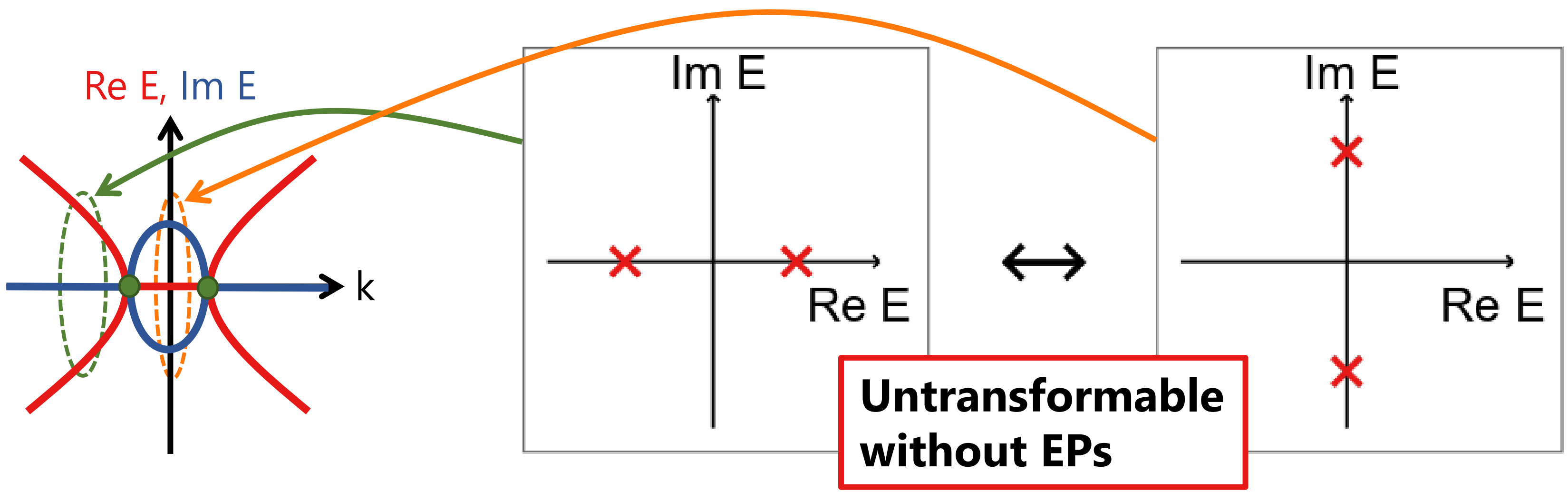
Exceptional points in 1D + symmetry

ex) PT (parity-time) symmetry

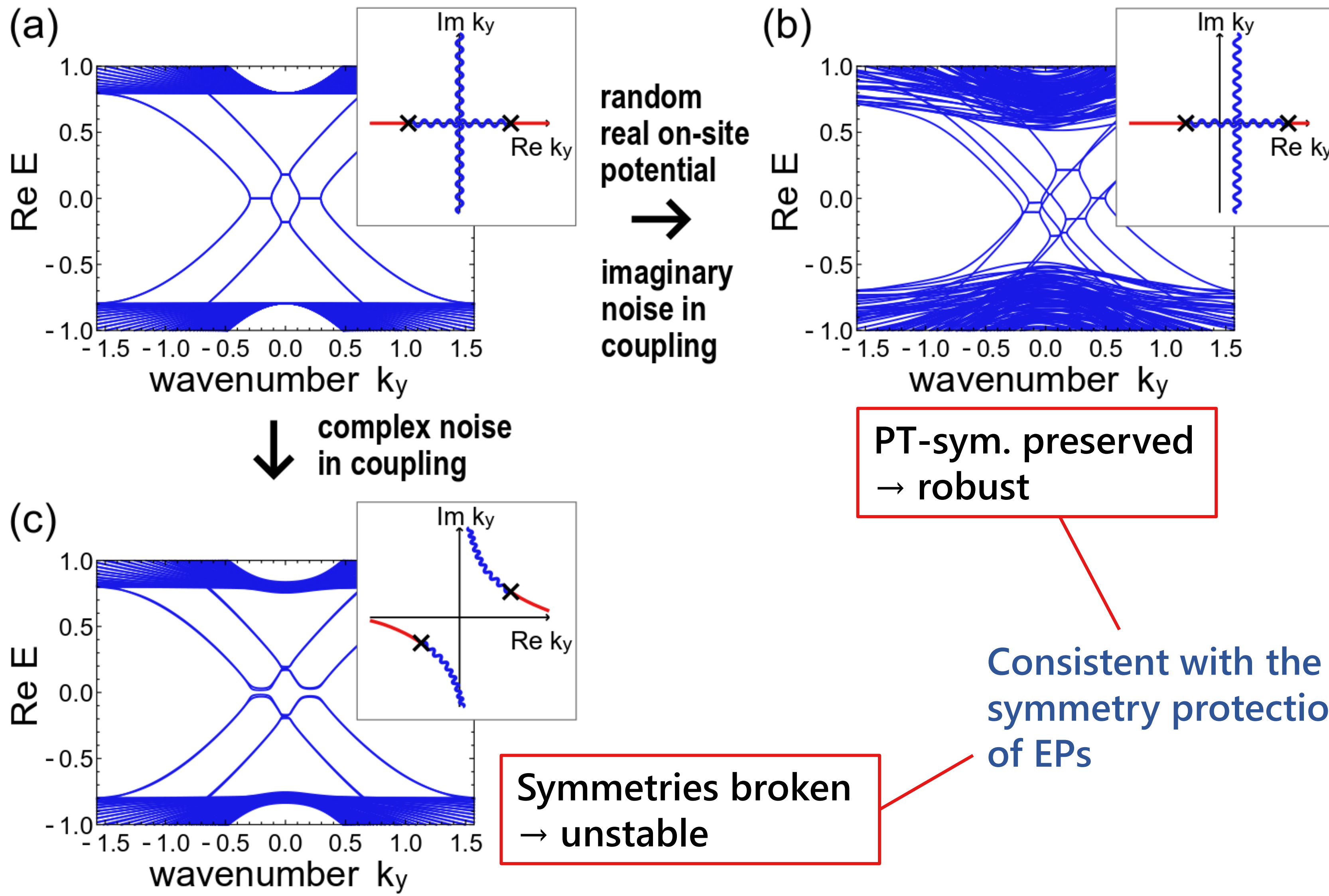
$$PTH(\boldsymbol{k})(PT)^{-1} = H^*(\boldsymbol{k})$$

→ Eigenvalues are real or pairs of complex conjugates.

→ EPs are protected!



Robustness Against the Disorder



PT-sym. preserved
→ robust

Consistent with the symmetry protection of EPs

Symmetries broken
→ unstable

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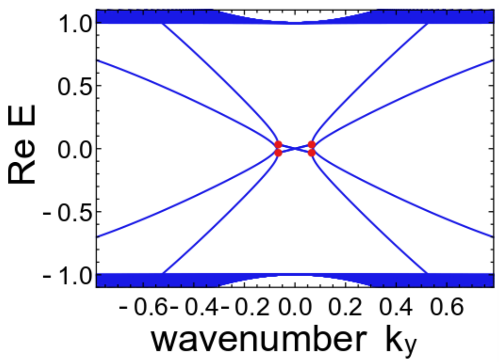
Remained Problems on Topological Lasers

Q1. Propagation of lasing wave packets
without judicious gain

→ Without need of the knowledge
of boundary configurations

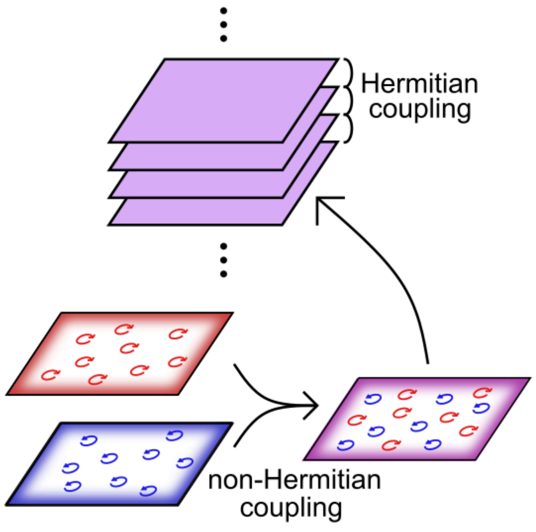
A1. Modified model of exceptional edge modes!



Q2. Topological lasers without
both juditicious gain and symmetries

→ More robust topological lasers

A2. Extension of exceptional edge modes
to 3D systems!



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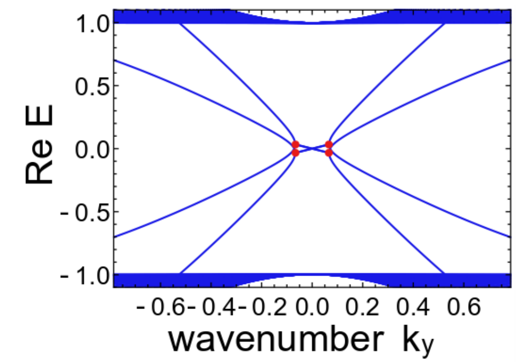

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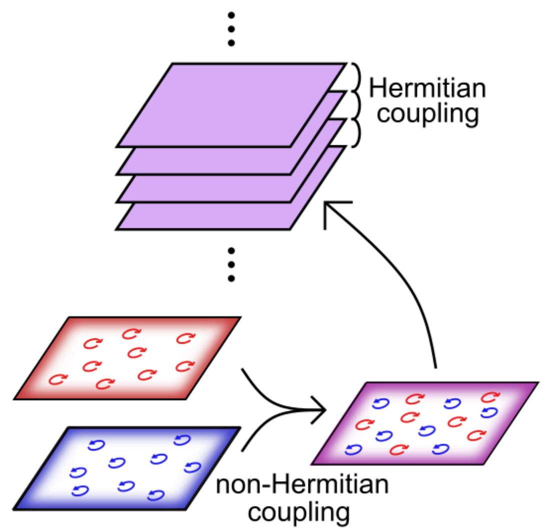
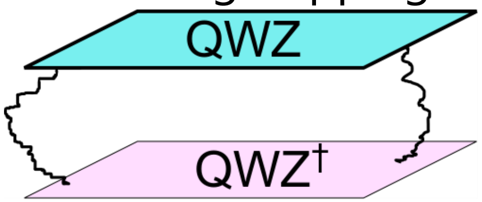


Application to 2D Topological Laser

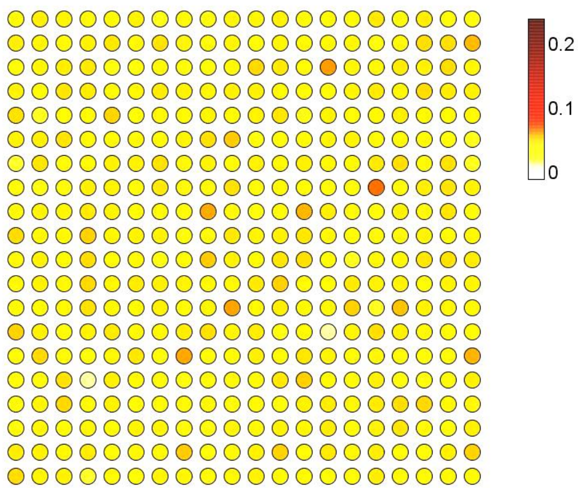
Hamiltonian

$$\begin{pmatrix} 2H_{\mathrm{QWZ}} & i\beta\sigma_x \\ i\beta'\sigma_x & H^*_{\mathrm{QWZ}} \end{pmatrix}$$

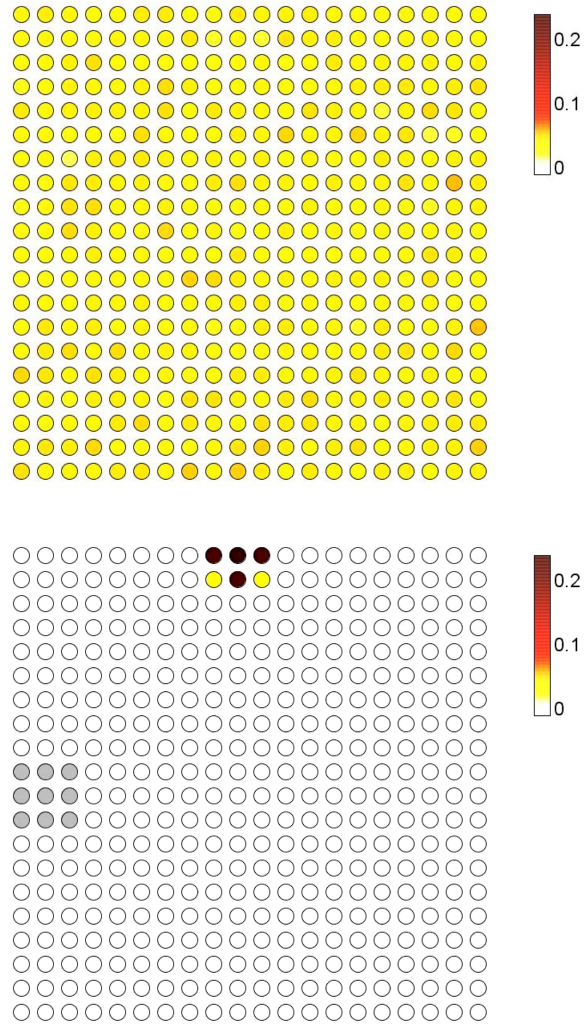
Strong hopping



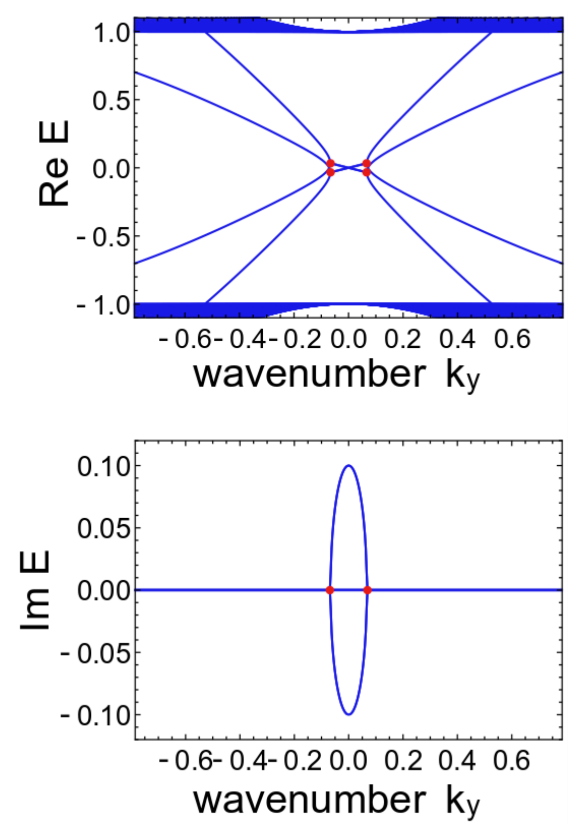
$\beta = \beta' = 0$



$\beta, \beta' > 0$



Edge band structure



→ Satisfying prerequisites for a topological laser

Application to 2D Topological Laser

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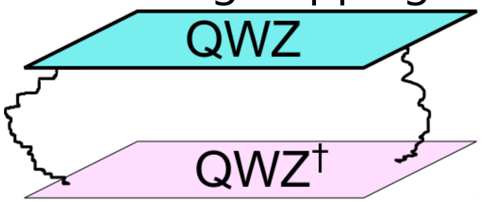

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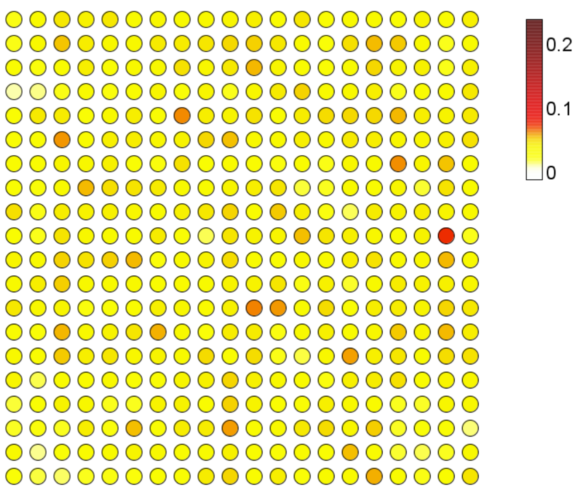

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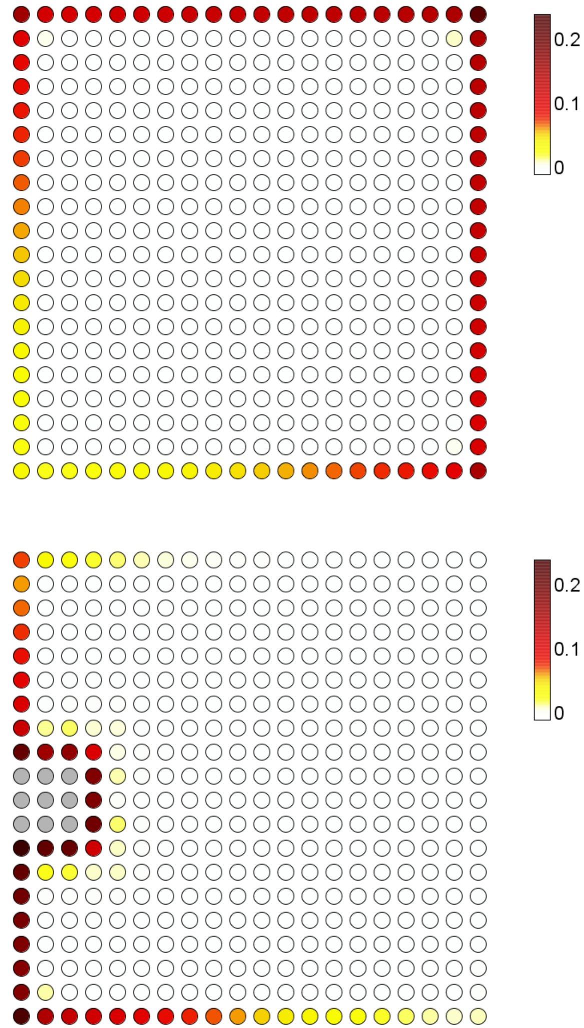

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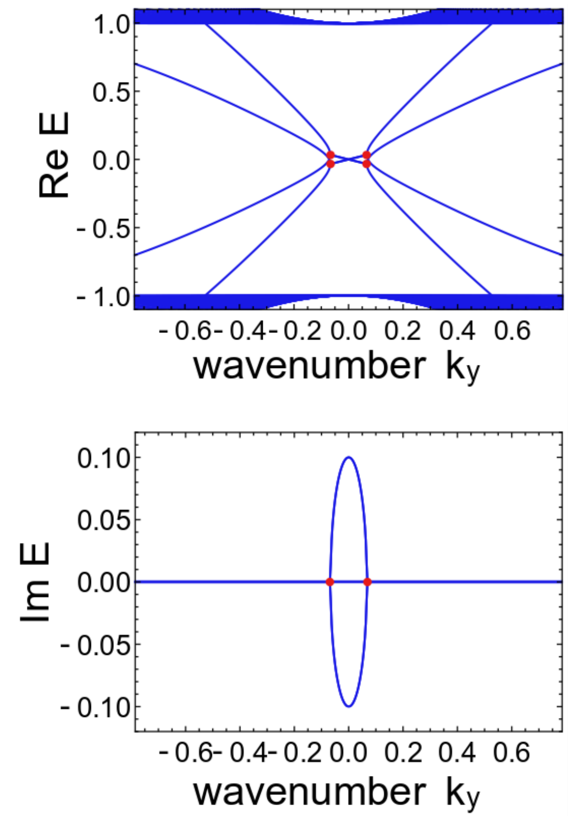

→ Satisfying prerequisites for a topological laser

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→ Without need of the knowledge
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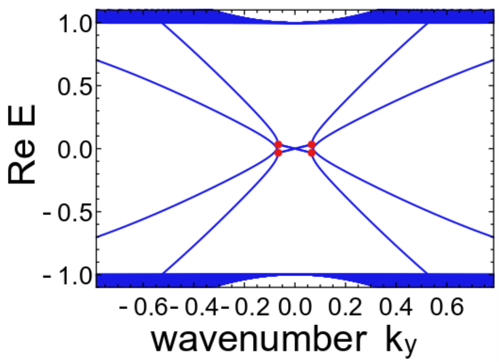
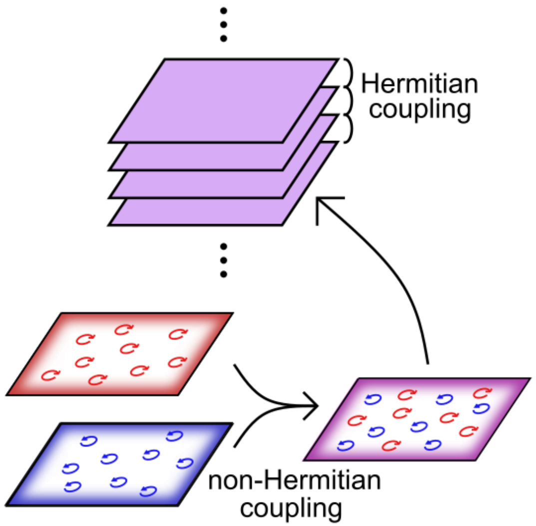


Q2. Topological lasers without
both juditicious gain and symmetries

→ More robust topological lasers

**A2. Extension of exceptional edge modes
to 3D systems!**



Construction Procedure of Three-Dimensional Topological Laser

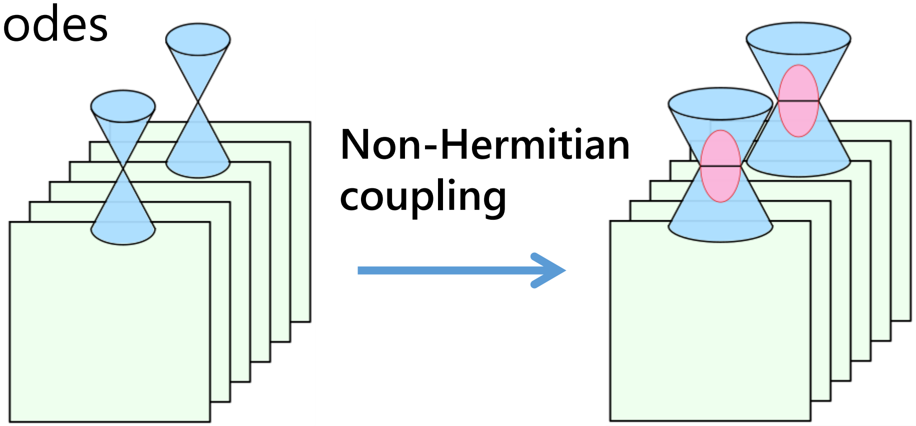
No symmetry, 3D → (Line-gap) topology is trivial.

(cf. K. Kawabata *et al.*, PRX 9, 041015 (2019).)

↕

Exceptional points in boundary bands (2D) are stable.

→ extension of exceptional edge modes



Weak topological insulator

(accumulation of
quantum spin Hall systems)

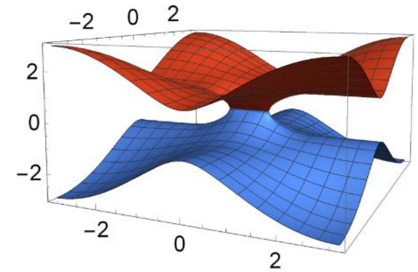
→ two Dirac cones

+

Non-Hermitian spin coupling

→ splitting into exceptional points

→ large imaginary parts of eigenvalues (⇒amplification)



cf. One pair of EPs
→ unstable under OBC

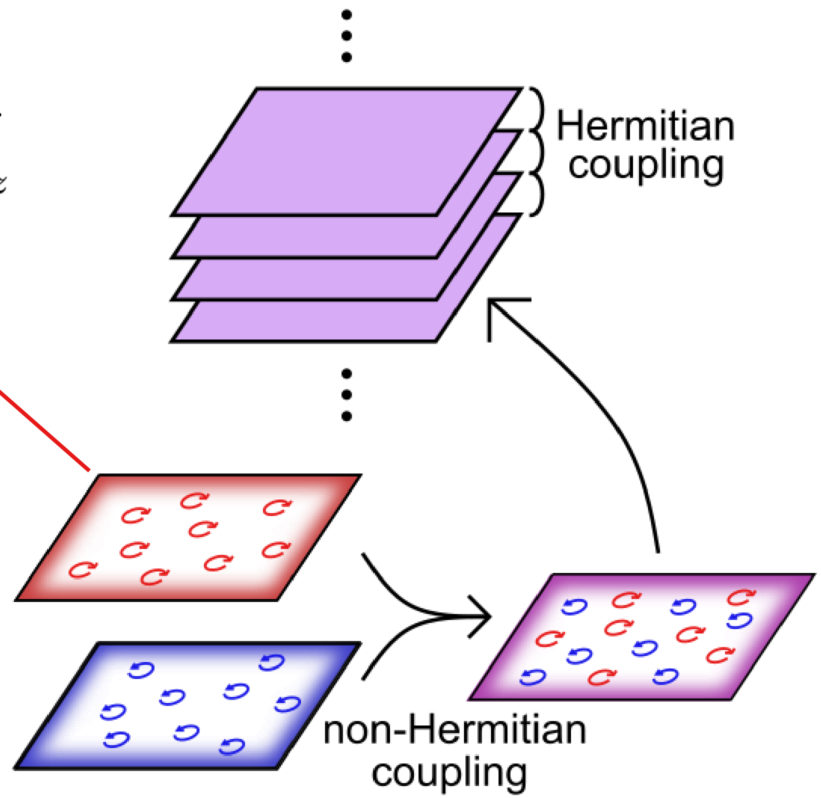
Toy Model: Weak Topological Insulator + Non-Hermitian Coupling

1. Chern insulator model
   X. QI, Y. S. Wu, S. C. Zhang PRB 74, 085308 (2006).
   $$H(\mathbf{k}) = (u + \cos k_x + \cos k_y)\sigma_z + \sin k_x \sigma_x + \sin k_y \sigma_y$$

2. Coupling with its time-reversal counterpart
   (quantum spin Hall (QSH) system)

3. Non-Hermitian couplings

4. Accumulation



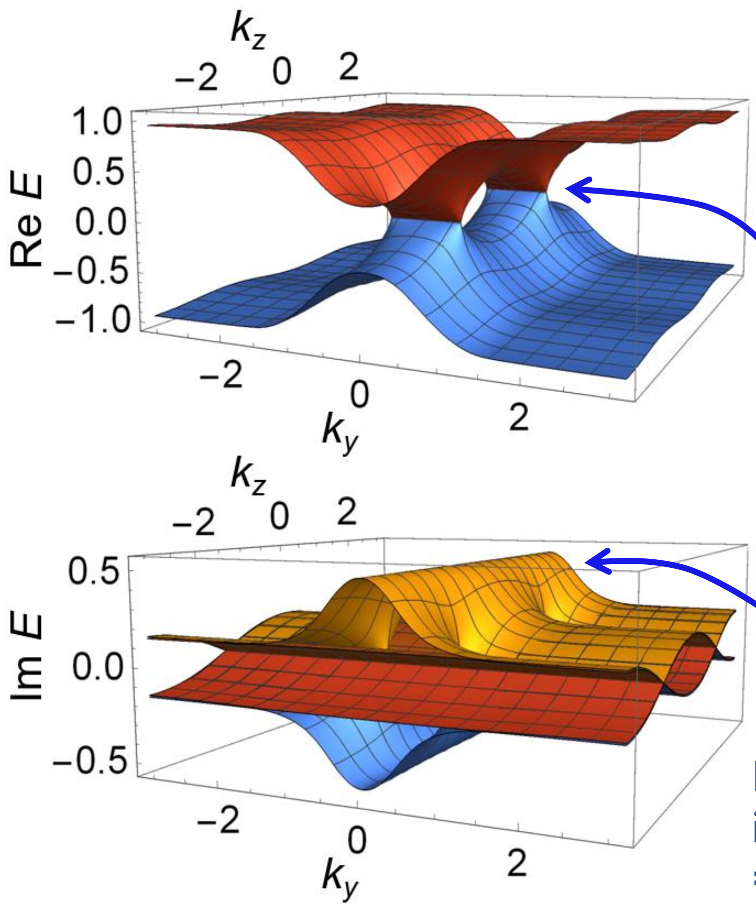
$$H(\mathbf{k}) = (u + \cos k_x + \cos k_y) I \otimes \sigma_z + \sin k_x \sigma_z \otimes \sigma_x + \sin k_y I \otimes \sigma_y + (c \sin k_z + i\gamma)\sigma_y \otimes \sigma_x$$

QSH system | Interlayer coupling | Non-Hermitian coupling

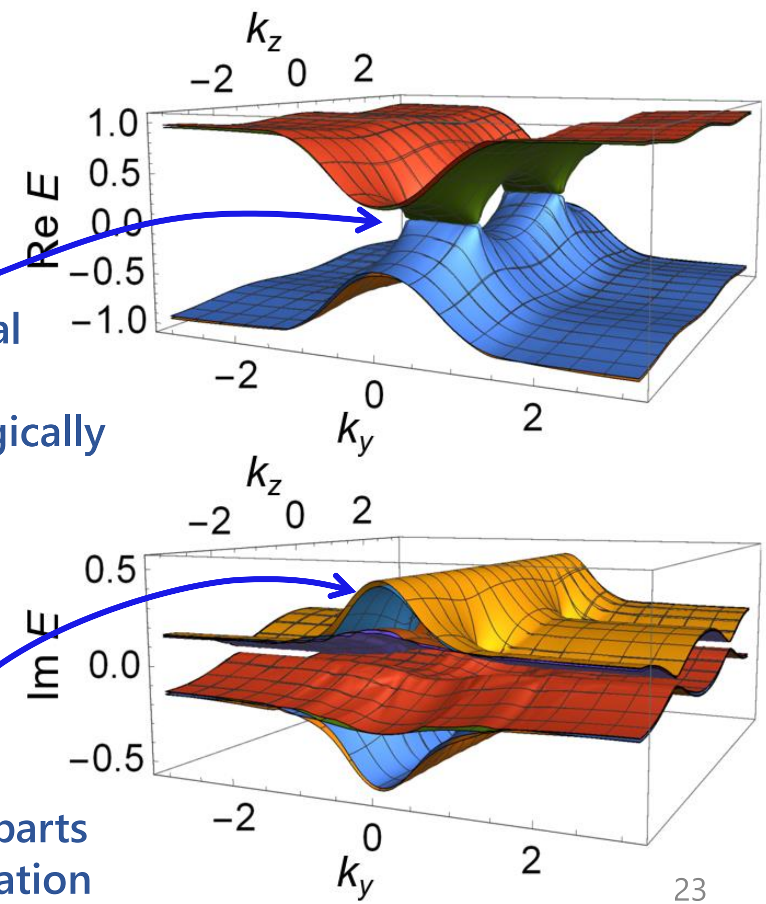
Band Structure of the Surface Modes

$$H(\mathbf{k}) = (u + \cos k_x + \cos k_y) I \otimes \sigma_z + \sin k_x \sigma_z \otimes \sigma_x + \sin k_y I \otimes \sigma_y + (c \sin k_z + i\gamma)\sigma_y \otimes \sigma_x$$

w/o noise



w/ noise



Exceptional points
⇒ topologically protected

Large imaginary parts
⇒ amplification

Nonzero Group Velocity: Transfer of Lasing Wave Packets

Asymmetry between (effective) spins

→ **Nonzero group velocity → Transfer of lasing wave packets**

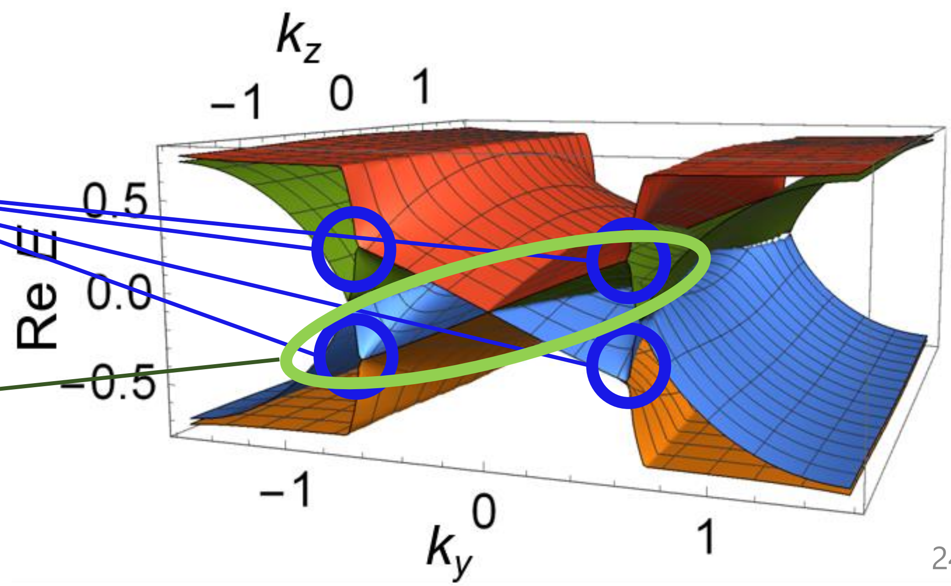
(cf. KS, Y. Ashida, and T. Sagawa, Nat. Commun. 11, 5745 (2020).)

$$
\begin{aligned}
H(\mathbf{k}) \quad = \quad & (u+\cos k_x+\cos k_y)\,(aI\otimes\sigma_z+\underline{b\sigma_z\otimes\sigma_z}) \\
& +\sin k_x\,(\underline{bI\otimes\sigma_x}+a\sigma_z\otimes\sigma_x) \\
& +\sin k_y\,(aI\otimes\sigma_y+\underline{b\sigma_z\otimes\sigma_y})+(c\cos k_z+i\gamma)\sigma_y\otimes\sigma_x.
\end{aligned}
$$

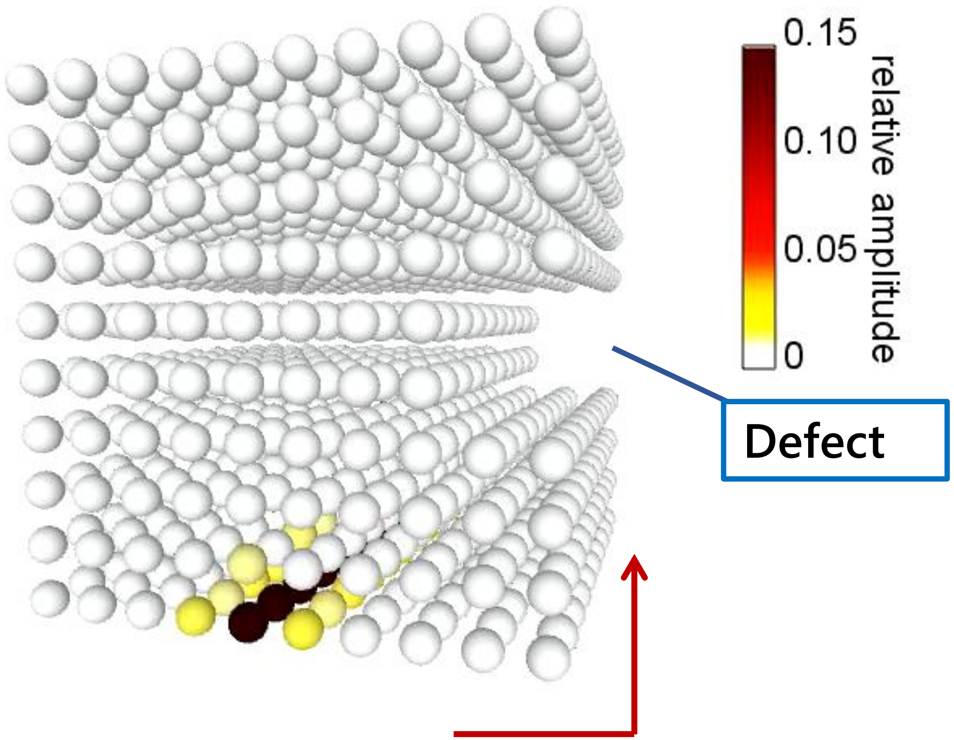
b: **strength of asymmetry**

Protection
by **exceptional points**

Boundary modes
with **nonzero group velocity**



Real-Space Dynamics

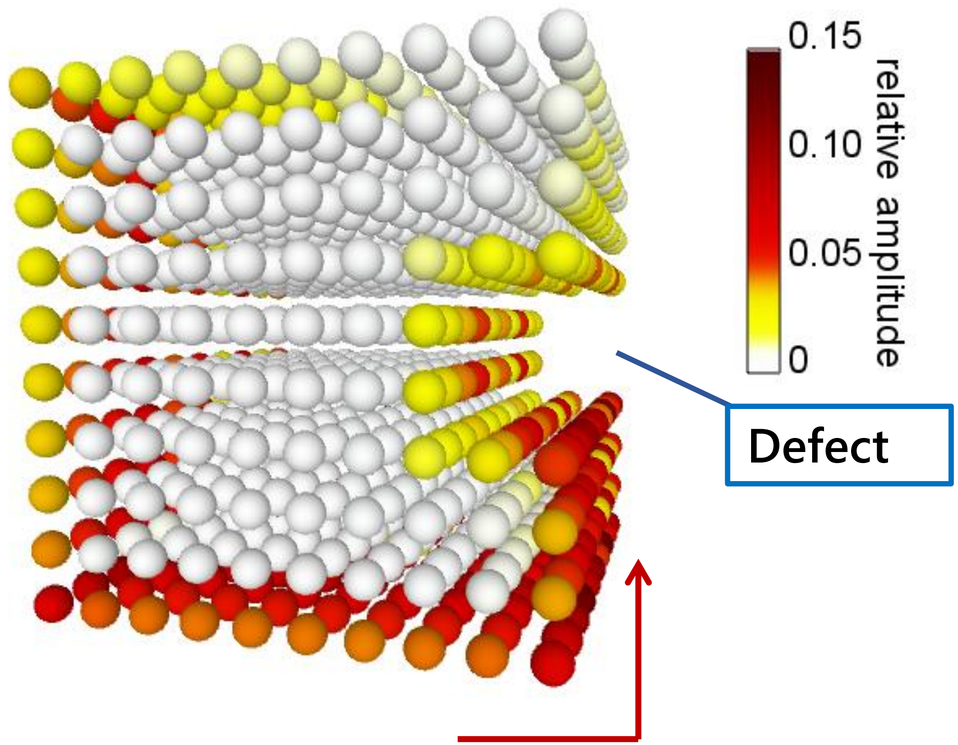


Lasing wave packets along the edge

・Nonzero group velocity

・Backscattering-free at the defect
= Robustness against disorders at the boundaries

Real-Space Dynamics



Lasing wave packets along the edge

・Nonzero group velocity

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= Robustness against disorders at the boundaries

Possible Optical Setup Using Ring-Resonator Arrays

Ring-resonator arrays

⇒ quantum spin Hall systems

- Effective spin: clockwise and counterclockwise modes
- Hopping: via evanescent light
- Artificial gauge field: phase-difference of propagating light

cf. M. Hafezi *et al.* Nat. Photonics 7, 1001 (2013).

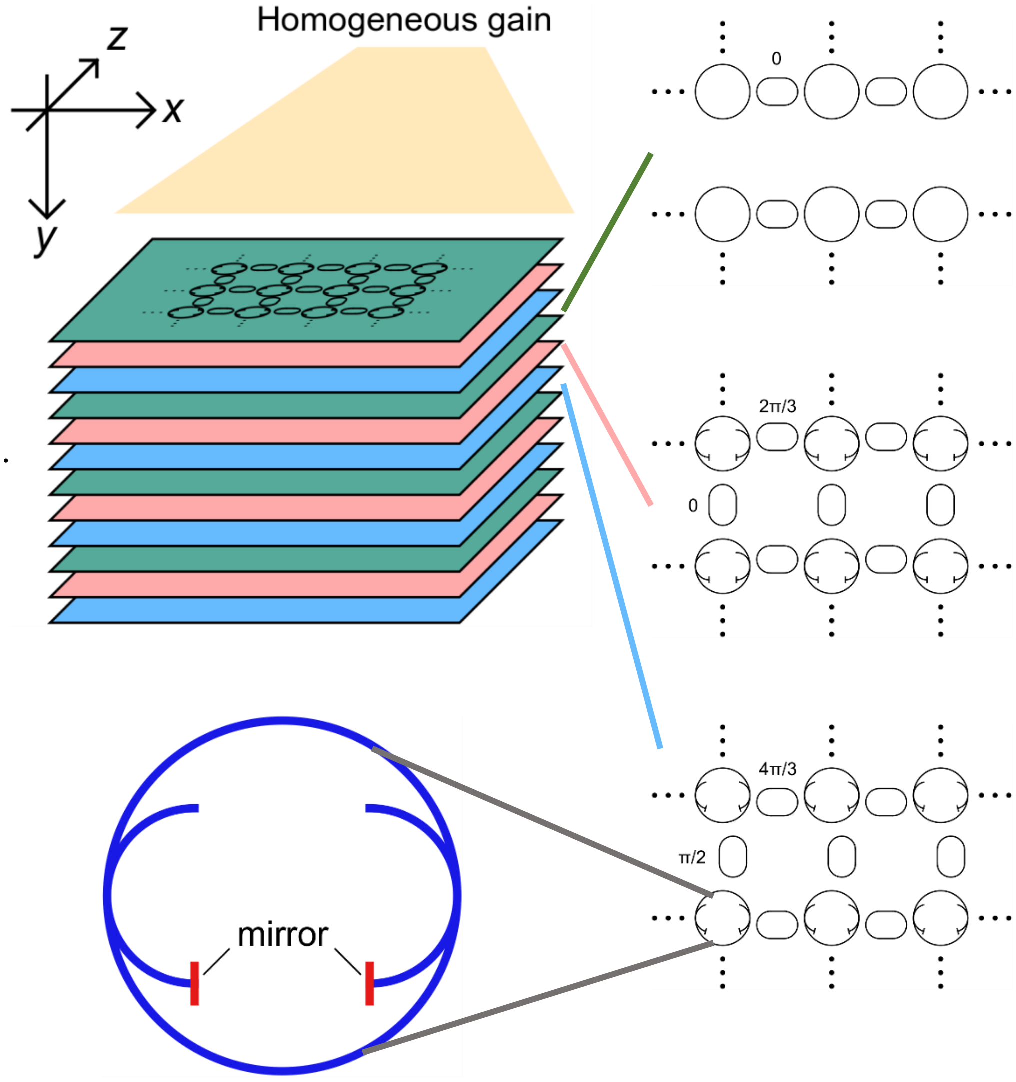
Accumulation

⇒ Counterpart of weak topological insulator

Waveguides with mirrors

⇒ Non-Hermitian coupling
(diminishing imbalance between CW and CCW modes)

cf. A. Y. Song *et al.* PRL 125, 033603 (2020).

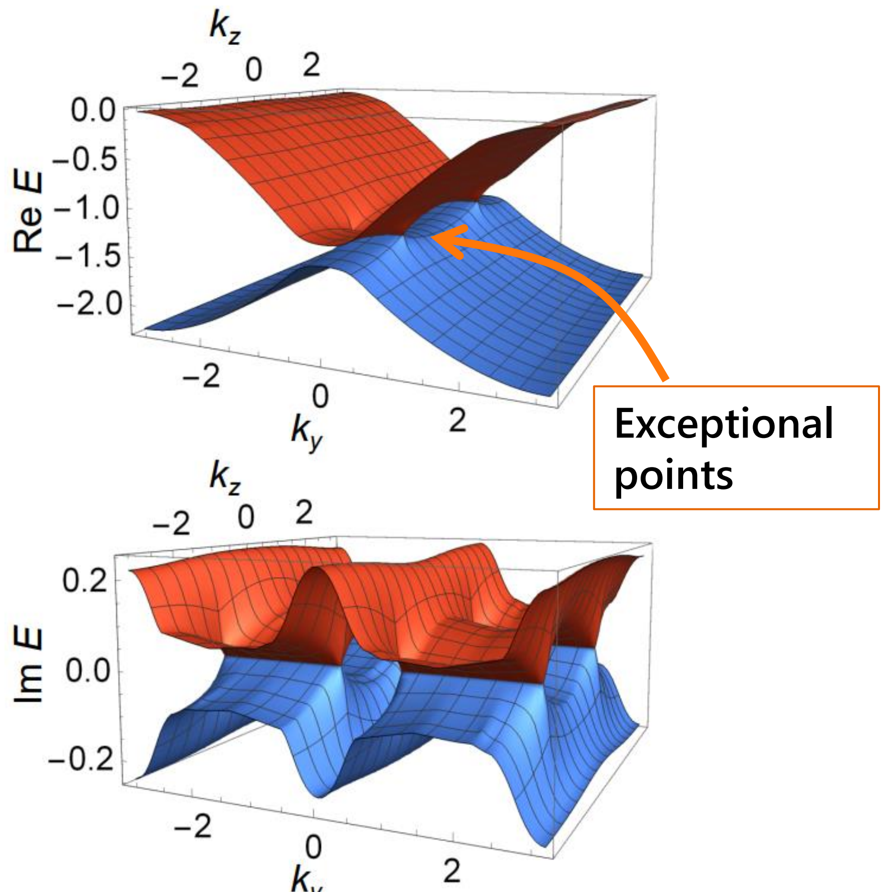


Numerical Results: Exceptional Surface Modes in Proposed Setup

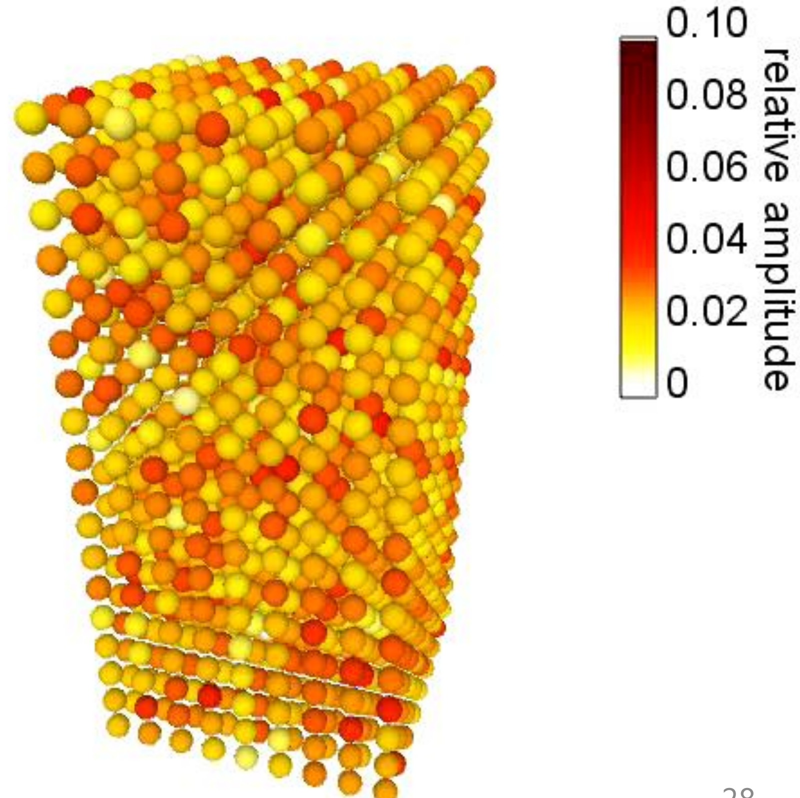
$$H = \sum_{\mathbf{r},s=\pm}(a_x e^{is\phi y}\hat{c}^\dagger_{\mathbf{r},s}\hat{c}_{\mathbf{r}+\mathbf{e}_x,s} + a_y \hat{c}^\dagger_{\mathbf{r},s}\hat{c}_{\mathbf{r}+\mathbf{e}_y,s}) + \text{H.c.}$$

$$+\sum_{\mathbf{r}}[a_z\sin(\phi y)(\hat{c}^\dagger_{\mathbf{r},+}\hat{c}_{\mathbf{r}+\mathbf{e}_z,-} + \hat{c}^\dagger_{\mathbf{r},-}\hat{c}_{\mathbf{r}+\mathbf{e}_z,+}) + \text{H.c.} + i\beta\sin(\phi y)(\hat{c}^\dagger_{\mathbf{r},+}\hat{c}_{\mathbf{r},-} + \hat{c}^\dagger_{\mathbf{r},-}\hat{c}_{\mathbf{r},+})]$$

Dispersion relation



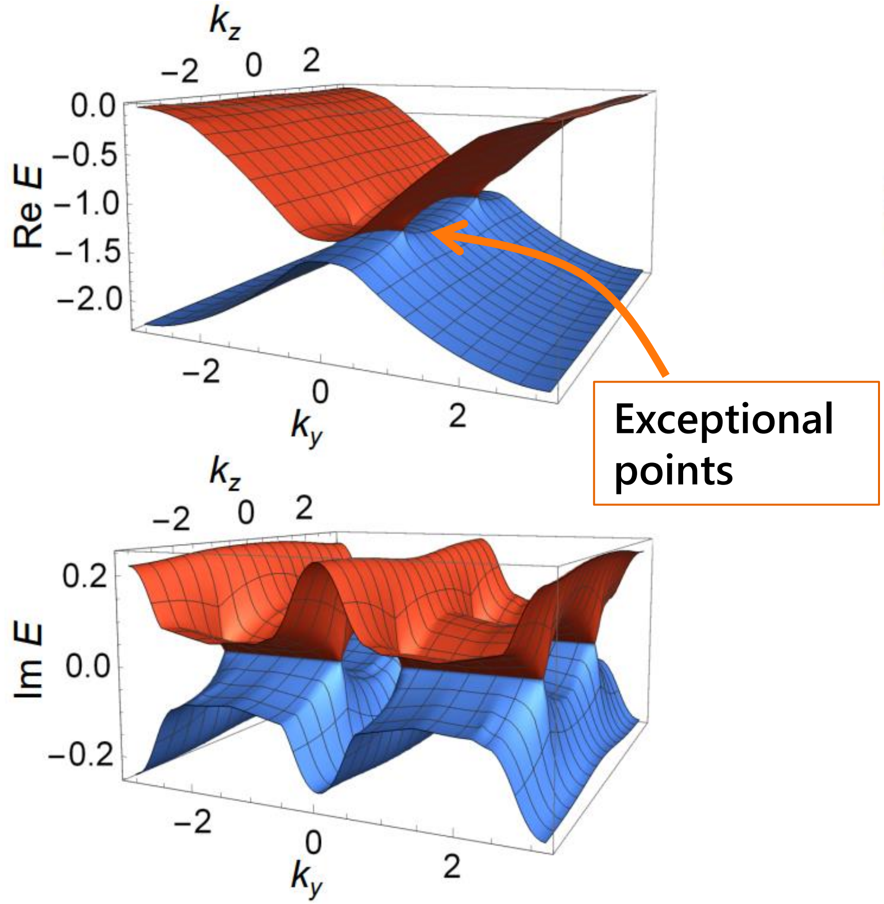
Real-space dynamics



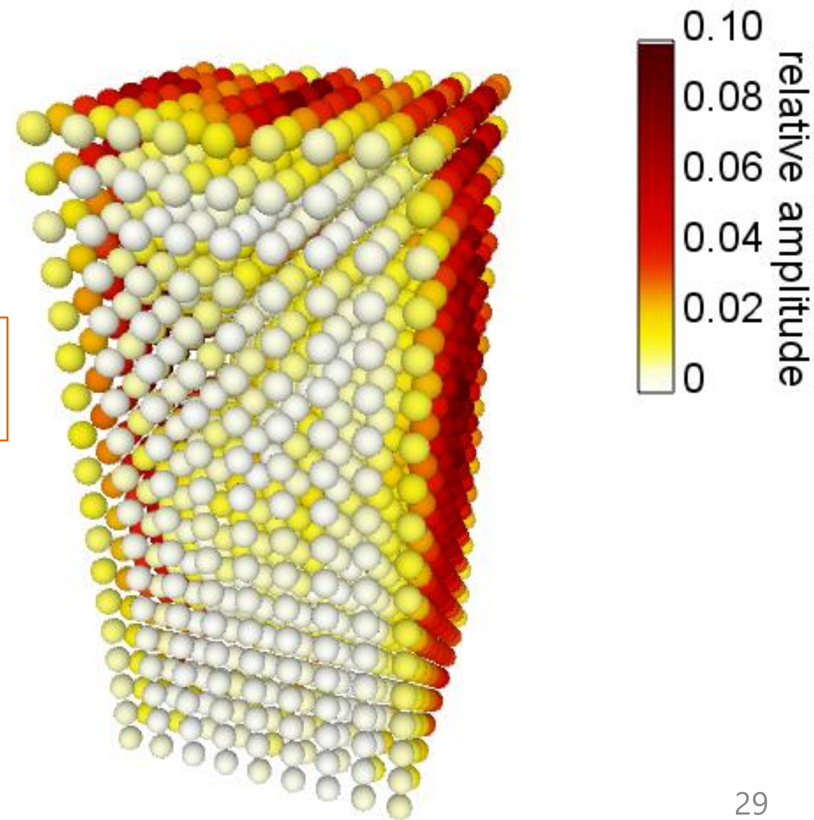
Numerical Results: Exceptional Surface Modes in Proposed Setup

$$H = \sum_{\mathbf{r},s=\pm} (a_x e^{is\phi y} \hat{c}^\dagger_{\mathbf{r},s} \hat{c}_{\mathbf{r}+\mathbf{e}_x,s} + a_y \hat{c}^\dagger_{\mathbf{r},s} \hat{c}_{\mathbf{r}+\mathbf{e}_y,s}) + \text{H.c.}$$
$$+ \sum_{\mathbf{r}} [a_z \sin(\phi y)(\hat{c}^\dagger_{\mathbf{r},+} \hat{c}_{\mathbf{r}+\mathbf{e}_z,-} + \hat{c}^\dagger_{\mathbf{r},-} \hat{c}_{\mathbf{r}+\mathbf{e}_z,+}) + \text{H.c.} + i\beta \sin(\phi y)(\hat{c}^\dagger_{\mathbf{r},+} \hat{c}_{\mathbf{r},-} + \hat{c}^\dagger_{\mathbf{r},-} \hat{c}_{\mathbf{r},+})]$$

Dispersion relation



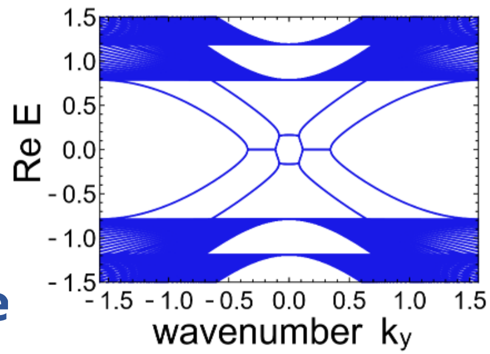
Real-space dynamics



Summary

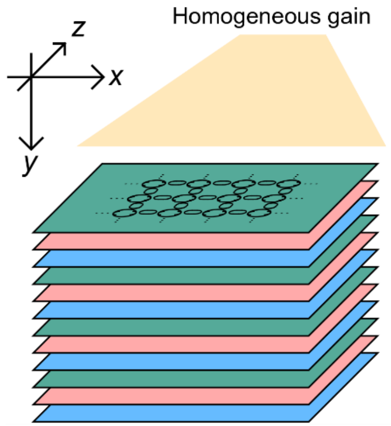
Exceptional edge modes

→ Protected by **exceptional points**



- Robust even with a topologically trivial bulk
  → **Breakdown of the bulk-edge correspondence**

Applications to topological laser



- Without judicious gains
- In 3D, no needs of symmetry protections
- Proposal of photonic systems

Refs: KS, Y. Ashida, and T. Sagawa, Nat. Commun. **11**, 5745 (2020).
KS, Y. Ashida, and T. Sagawa, Phys. Rev. B 105, 235426 (2022).