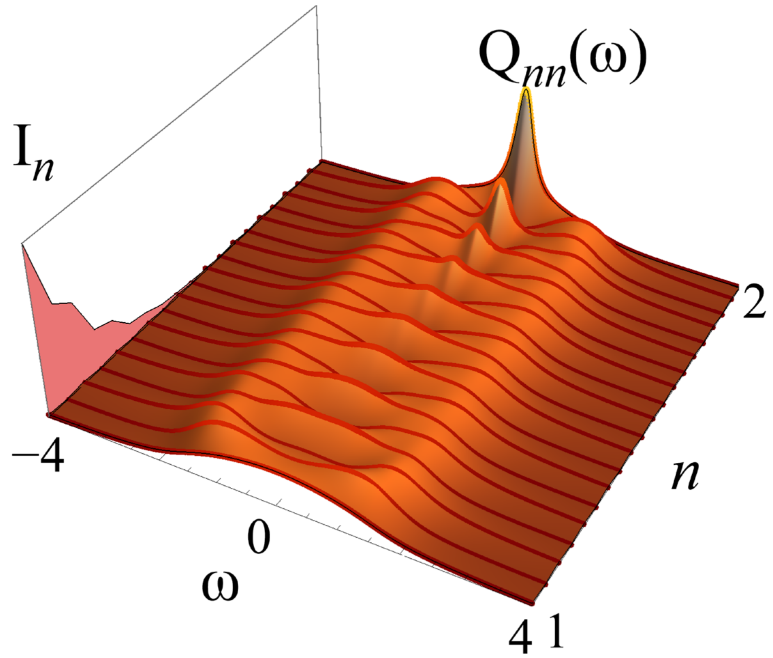


Causality constraints and their symmetry classification in passive devices

Henning Schomerus, Lancaster University & RIKEN

NH QM workshop, Tokyo University, 4 July 2022

NH effects

complex frequencies

exceptional points

nonorthogonal modes

NH skin effect

complications

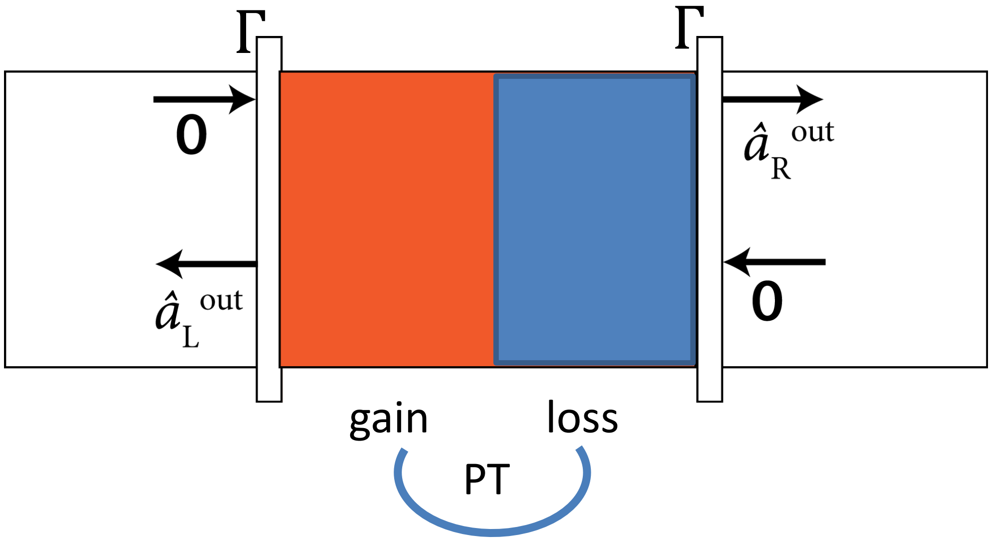
finite life times

enhanced sensitivity

instability

quantum noise

Example I: quantum noise



exact PT symmetric phase:

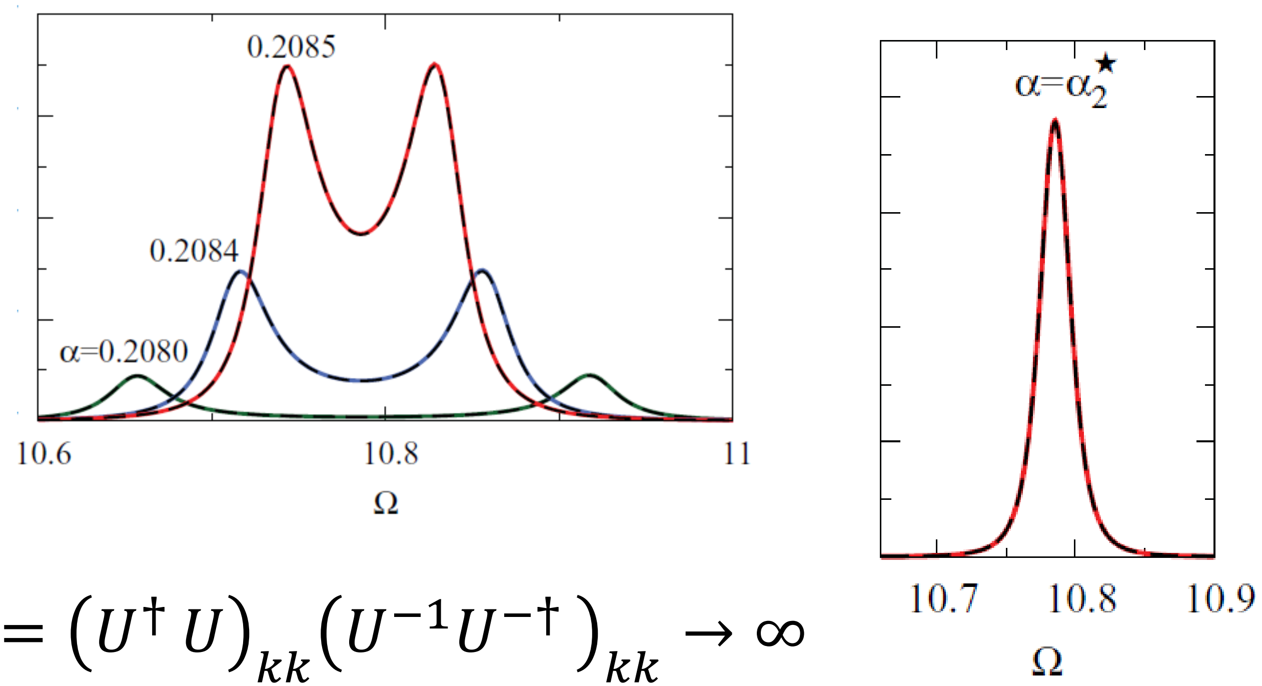
$I(\omega) = \mathrm{tr}\,(S^\dagger S - 1)/2\pi$
remains finite for $\Gamma \to 0$

HS, PRL 2010

EP to broken phase

$I(\omega) \to (\text{Lorentzian})^2$

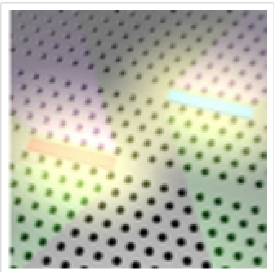
Yoo, Sim & HS, PRA 2011



Petermann factor $K_k = (U^\dagger U)_{kk}(U^{-1}U^{-\dagger})_{kk} \to \infty$

Example I: quantum noise

Optica Vol. 8, Issue 2, pp. 184-192 (2021) • https://doi.org/10.1364/OPTICA.412596



Observing exceptional point degeneracy of radiation with electrically pumped photonic crystal coupled-nanocavity lasers

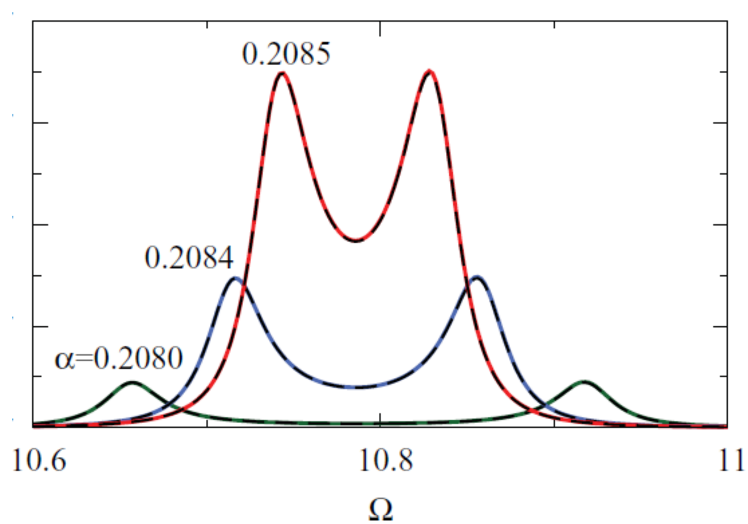
Kenta Takata, Kengo Nozaki, Eiichi Kuramochi, Shinji Matsuo, Koji Takeda, Takuro Fujii, Shota Kita, Akihiko Shinya, and Masaya Notomi

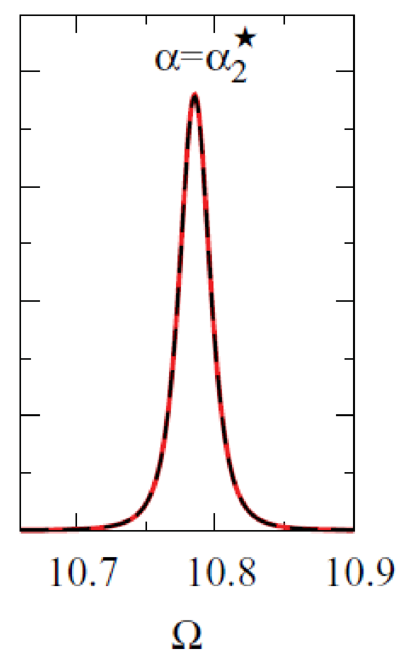
Furthermore, we find experimentally and confirm theoretically the peculiar squared Lorentzian emission spectrum very near the exact EP, which

EP to broken phase

$I(\omega) \to (\text{Lorentzian})^2$

Yoo, Sim & HS, PRA 2011

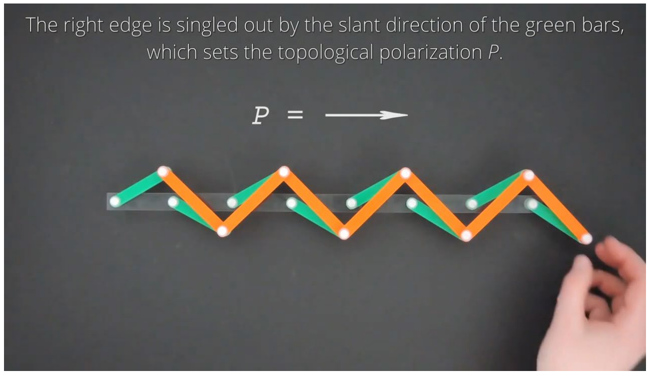




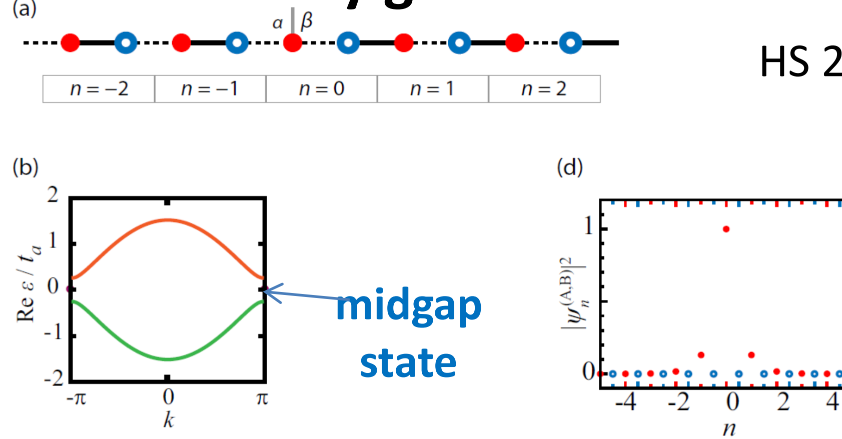
Petermann factor $K_k = \left(U^\dagger U\right)_{kk} \left(U^{-1} U^{-\dagger}\right)_{kk} \to \infty$

Example II: mode selection

topological zero mode



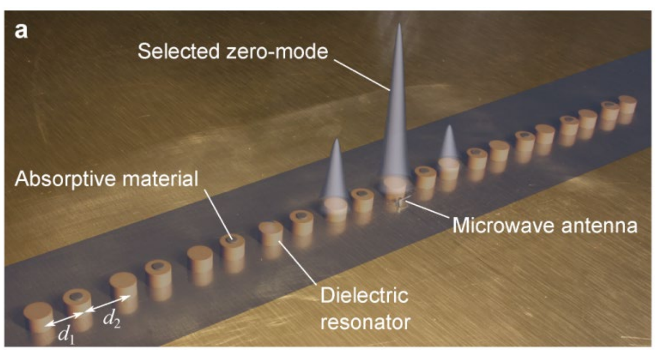
selected by gain and loss



HS 2013

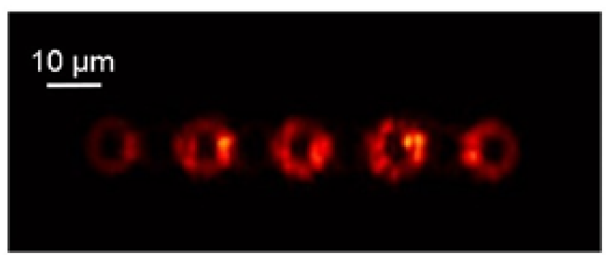
$H = -ZH^{*}Z, \mathrm{tr}\, Z = 1$ (C symmetry)

microwaves



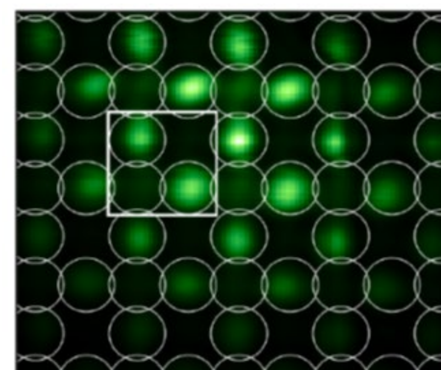
with Poli et al 2015

lasers



with Zhao et al 2018

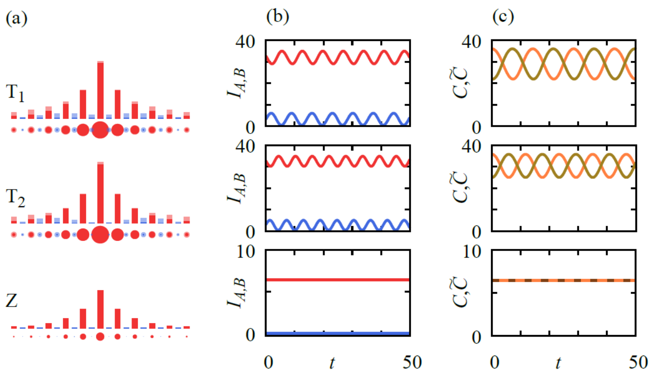
excitons



with Whittaker et al 2018

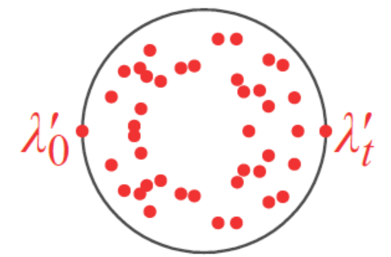
Example II: mode selection

Nonlinear symmetry



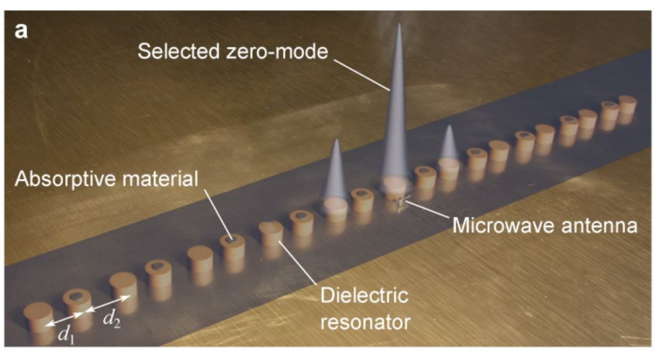
Malzard, Cancellieri & HS 2018

Symmetry-protected Topological excitations oscillations



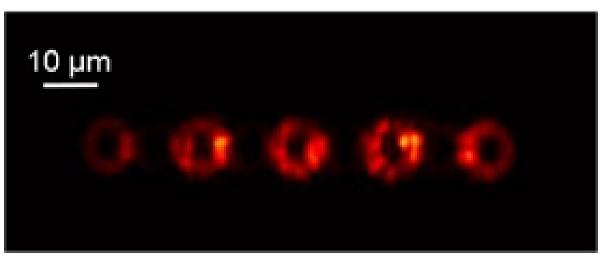
Goldstone modes:
U(1) and time translation

microwaves



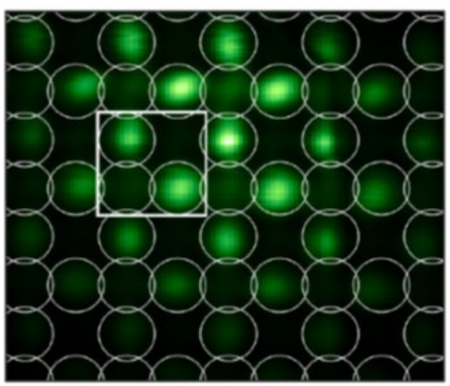
with Poli et al 2015

lasers



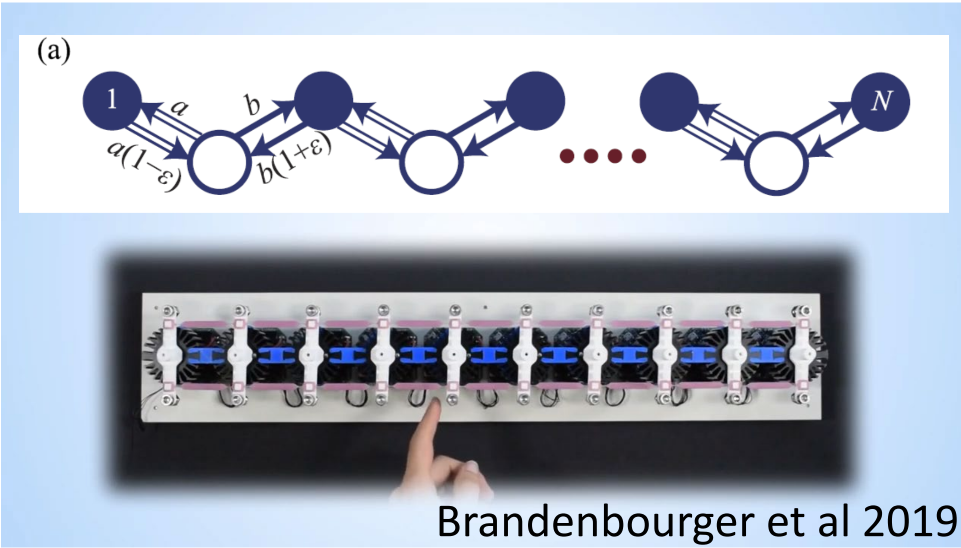
with Zhao et al 2018

excitons

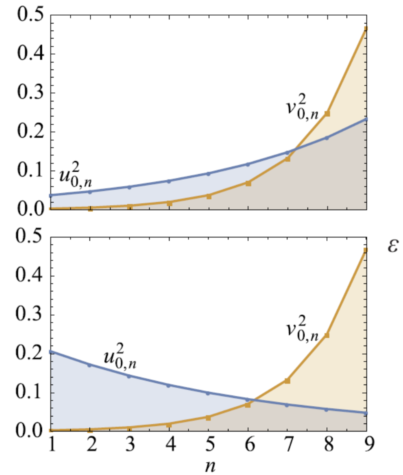


with Whittaker et al 2018

Example III: skin effect

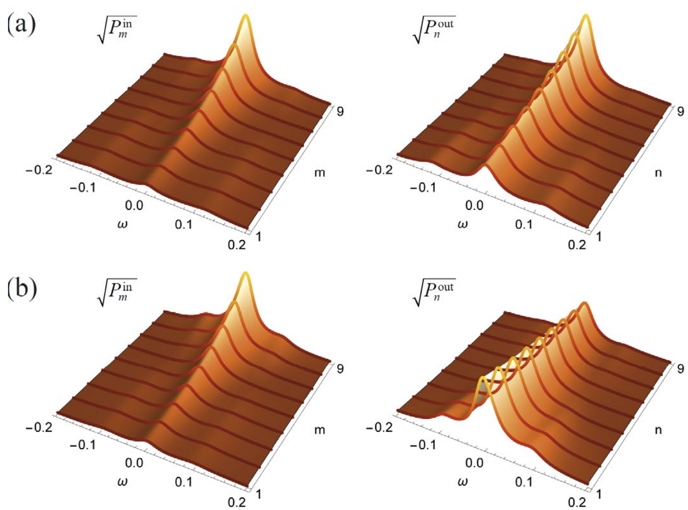


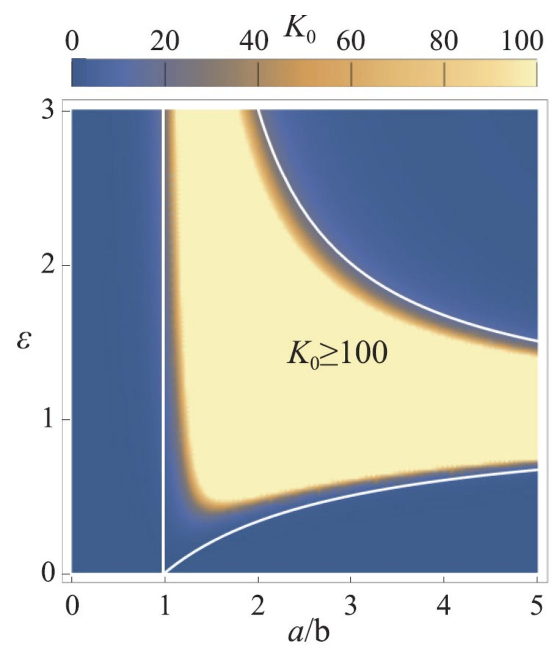
zero mode relocalises



HS, PRR 2020

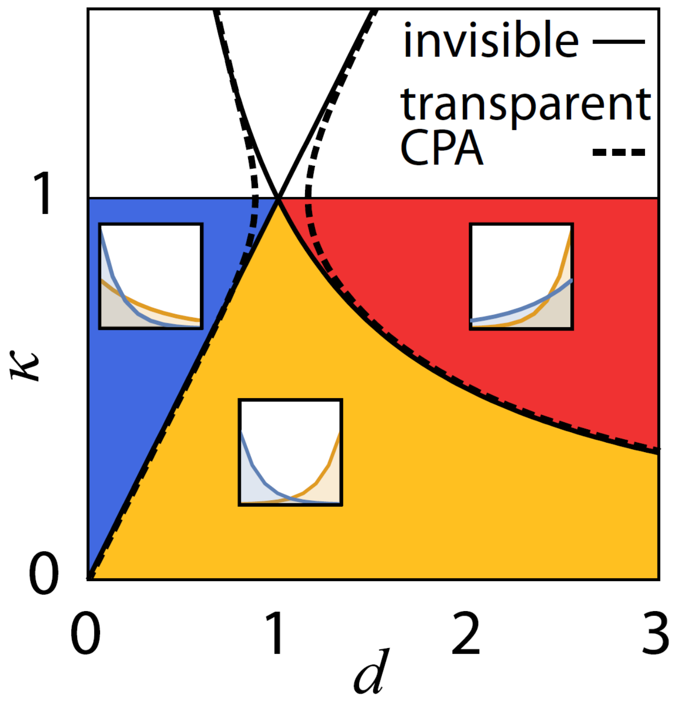
transition to directed amplification





also in transport effects

- ***reflectionless transport depending on topological phase***
- ***invisibility coinciding at skin-effect phase transition***
- ***CPA and one-sided transparency***



Ghaemi-Dizicheh & HS, PRA 2021

Here: passive systems

- physical limits from causality
- symmetry constraints
- visibility of effects:
  EPs, skin effect, edge states

Causality

recall output intensity $I(\omega) = \text{tr}\,(S^\dagger S - 1)/2\pi$

passive system: $1 - S^\dagger S$ is positive definite

generic wide-band limit: $S(\omega) = \frac{1 - i\Gamma\mathcal{G}\,(\omega)}{1 + i\Gamma\mathcal{G}\,(\omega)}$ with $\mathcal{G}(\omega) = \frac{1}{\omega - H}$

$\Rightarrow 1 - S^\dagger S = 2\,\Gamma\,\mathrm{Q}$ with $\mathrm{Q} = 2\mathcal{G}^\dagger(\omega + i\Gamma)\,(\gamma - F)\mathcal{G}(\omega + i\Gamma)$

- time-delay op., gives density of states $\rho(\omega) = \text{tr}\,Q/2\pi$
- $H = H_0 + iF - i\gamma$: nontrivial NH in F, background losses γ
- Q positive definite: causality threshold γ

causality > Lee-Wolfenstein (Wiersig 2019) > positive lifetimes

Symmetry classification

$H = H^*$	(TRS)	$\Rightarrow$	$F = -F^T$	(Majorana basis)
$H = H^T$	(reciprocal)	$\Rightarrow$	$F = F^*$	(TRS)
$H = PH^*P$	(PT)	$\Rightarrow$	$F = -PF^*P$	(charge conj, C)
$H = PH^\dagger P$	(PTT’)	$\Rightarrow$	$F = -PFP$	(chiral)
$H = -PH^*P$	(C)	$\Rightarrow$	$F = PF^*P$	(gen TRS)
$H = -PH^\dagger P$	(CT’)	$\Rightarrow$	$F = PFP$	(parity)

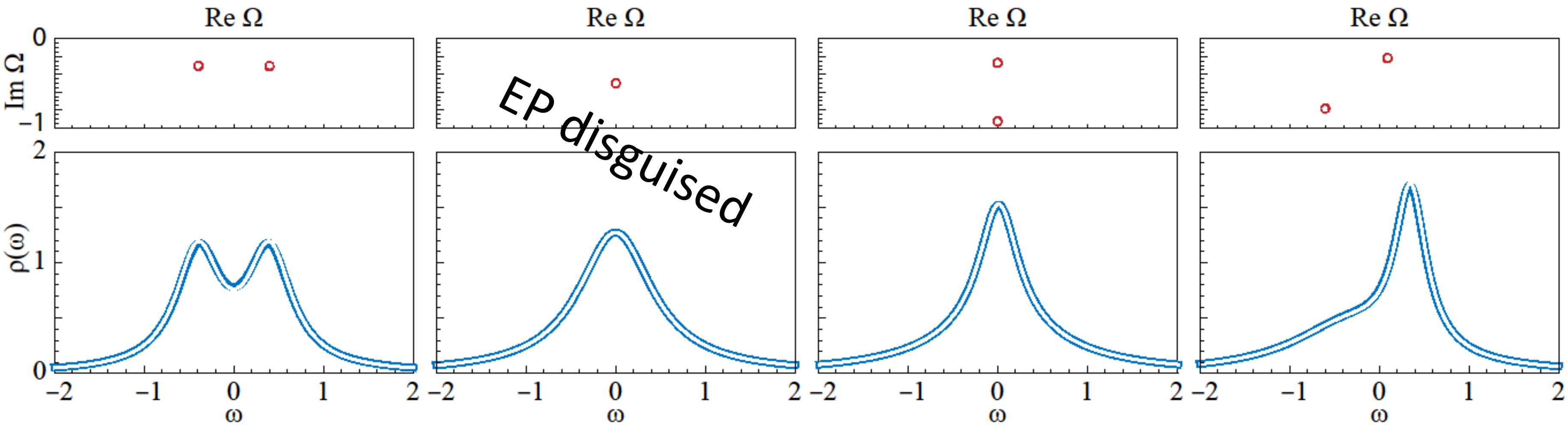
- systematic pairing of NH and H symmetry classes
- classes of H and iH differ

Visibility of EPs

EP normal form $H = \begin{pmatrix} a - i\gamma & b \\ c & -a - i\gamma \end{pmatrix} \Rightarrow F = \frac{1}{2i}\begin{pmatrix} a - a^* & b - c^* \\ c - b^* & a^* - a \end{pmatrix}$

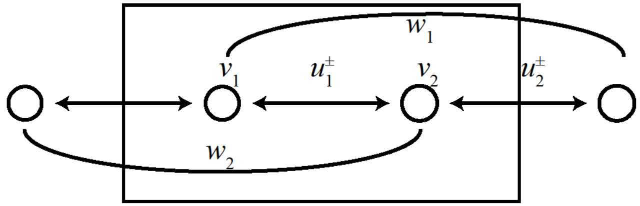
causality threshold $\gamma_c = \sqrt{(\mathrm{Im}\, a)^2 + |b - c^*|^2/4}$

EP $a^2 + bc = 0$: dos $\rho^{EP}(\omega) = \frac{1}{\pi}\frac{|b|+|c|}{\omega^2 + (|b|+|c|)^2/4}$ is a *simple* Lorentzian!



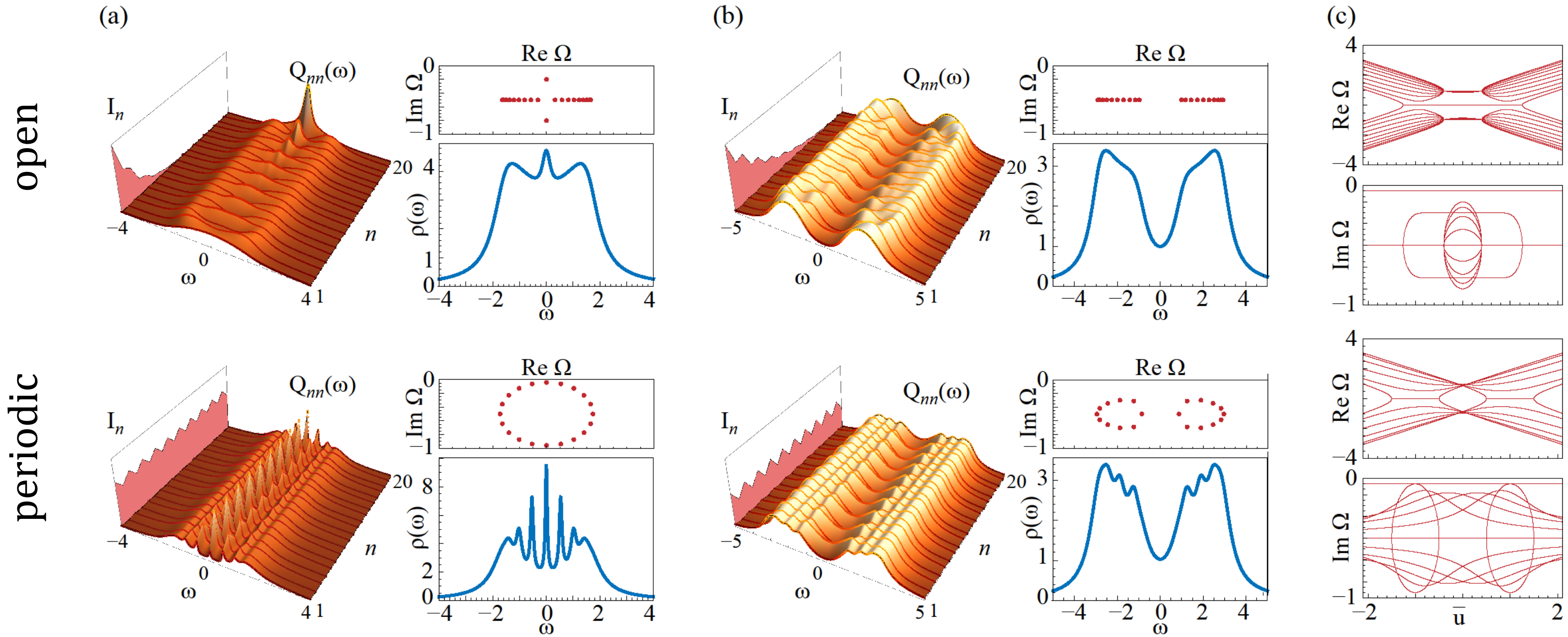
Visibility of skin effect and edge states

nonreciprocal dimer chain



$$H(k) = \begin{pmatrix} v_1 + 2w_1 \cos k & u_1^- + u_2^+ e^{-ik} \\ u_1^+ + u_2^- e^{ik} & v_2 + 2w_2 \cos k \end{pmatrix}$$

$$\Rightarrow \; F(k) \Rightarrow \gamma_c = \sqrt{\Delta v^2 + (|\Delta \mathrm{u}_1| + |\Delta \mathrm{u}_2|)^2}$$



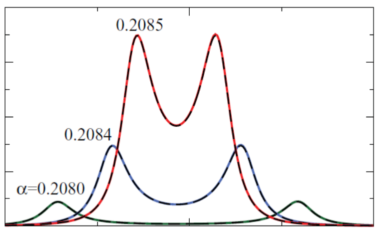
⇒ skin effect disguised, edge states visible

acknowledgements/references

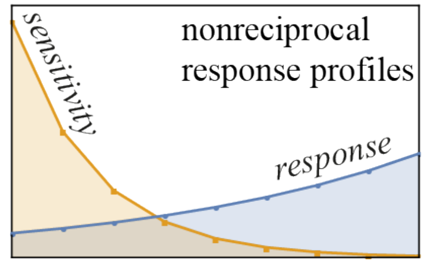
Intro:	
Quantum noise in PT exact phase:	HS, PRL **104**, 233601 (2010)
$I(\omega) \rightarrow (\text{Lorentzian})^2$ at EP:	G Yoo, H-S Sim & HS, PRA **84**, 063833 (2011)
Topological mode selection theory:	HS, Opt Lett **38**, 1912-1914 (2013)
Microwave demonstration:	C Poli et al, Nat Commun **6**, 6710 (2015)
Topological laser:	H Zhao et al, Nat Commun **9**, 981 (2018)
Exciton condensate:	C Whittaker et al, PRL **120**, 097401 (2018)
Nonlinear extensions:	S Malzard et al, Opt Exp **26**, 22506 (2018)
	S Malzard & HS, New J Phys **20**, 063044 (2018)
Directed amplification & sensing:	HS, Phys Rev Research **2**, 013058 (2020)
Nonreciprocal transport signatures:	H Ghaemi-Dizicheh & HS, PRA **104**, 023515 (2021)
Causality constraints:	**HS, in preparation**

Summary: NH topology from gain & loss

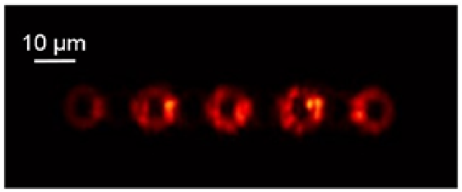
quantum noise



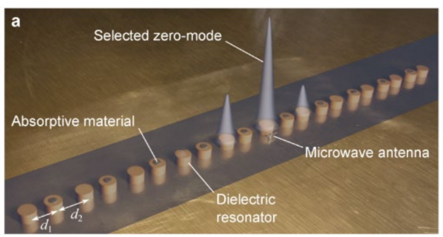
directed ampl.



⇒ lasers, sensors ...



mode selection



passive devices:

causality constraints ⇒ threshold losses

scattering theory ⇒ EP’s & NHSE disguised, edge states visible