Reservoir-assisted symmetry breaking and coalesced zero-energy modes in an open PT-symmetric SSH model

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Exceptional points: definition

An ***exceptional point*** **(EP)** is a branch point in parameter space at which two or more eigenstates *coalesce*

- Hamiltonian cannot be diagonalized at the EP

$$R^{-1}HR \sim \begin{pmatrix} \overline{E}_{EP} & c \\ 0 & \overline{E}_{EP} \end{pmatrix}$$

T. Kato, Perturbation Theory for Linear Operators (Springer, 1976)

G. Bhamathi and E. C. G. Sudarshan, Int. J. Mod. Phys. B **10**, 1531 (1996).

K. Kanki, S. Garmon, S. Tanaka, and T. Petrosky,
J. Math. Phys. **58**, 092101 (2017).

Exceptional points: definition

An ***exceptional point*** **(EP)** is a branch point in parameter space at which two or more eigenstates *coalesce*

- Hamiltonian cannot be diagonalized at the EP

- This is distinct from the usual concept of degeneracy in quantum systems

- Exceptional points can only appear in non-Hermitian systems

Exceptional points: Puiseux expansion

Eigenvalues in the vicinity of the exceptional points can be expanded in characteristic Puiseux expansion:

$$z_{1,2} = \bar{z}_{\mathrm{EP}} \pm \alpha\left(\varepsilon - \bar{\varepsilon}_{\mathrm{EP}}\right)^{1/2} + O(\varepsilon - \bar{\varepsilon}_{\mathrm{EP}})$$

coalesced eigenvalue

$\bar{\varepsilon}_{\mathrm{EP}}$ is a branchpoint in ε space

Exceptional points: order mismatch

Usually, the Jordan block dimension and the root in the Puiseux expansion will match; but not always.

Third order (EP3) :

$$R^{-1}HR \sim \begin{pmatrix} \bar{E}_{EP} & c_1 & 0 \\ 0 & \bar{E}_{EP} & c_2 \\ 0 & 0 & \bar{E}_{EP} \end{pmatrix}$$

expectation:

$$z_j = \bar{z}_{\mathrm{EP}} + \alpha_j \left(\varepsilon - \bar{\varepsilon}_{\mathrm{EP}}\right)^{1/3} + O\left(\left(\varepsilon - \bar{\varepsilon}_{\mathrm{EP}}\right)^{2/3}\right)$$

G. Demange and E.-M. Graefe, J. Phys. A: Math. Theor. **45**, 025303 (2012).

Exceptional points: order mismatch

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Third order (EP3) :

$$R^{-1}HR \sim \begin{pmatrix} \overline{E}_{EP} & c_1 & 0 \\ 0 & \overline{E}_{EP} & c_2 \\ 0 & 0 & \overline{E}_{EP} \end{pmatrix}$$

instead, we may have

$$z_{1,2} = \overline{z}_{\mathrm{EP}} \pm \alpha\left(\varepsilon - \overline{\varepsilon}_{\mathrm{EP}}\right)^{1/2} + O(\varepsilon - \overline{\varepsilon}_{\mathrm{EP}})$$

EP2-like expansion + Taylor series

G. Demange and E.-M. Graefe, J. Phys. A: Math. Theor. **45**, 025303 (2012).

Exceptional points: non-Hermitian systems

Exceptional points can only occur in non-Hermitian systems

- Open quantum systems (implicit non-Hermiticity)

  Non-Hermiticity is associated with the energy continuum that describes the environment (structured reservoir)

- Explicitly non-Hermitian systems

  Example: PT-symmetric, pseudo-Hermitian systems

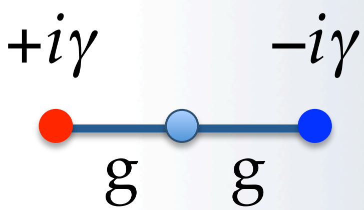
  <u>Coupled mode theory</u>: assumes relevant physics can be described in terms of a few key modes

PT-symmetric systems

<u>Parity-time (PT) symmetry</u>: studied as a potential replacement for Hermitian symmetry in quantum mechanics.

C. M. Bender, D. C. Brody and H. F. Jones,
Phys. Rev. Lett. **89**, 270401 (2002).

Simple PT-symmetric system:



γ : PT-parameter

Energy eigenvalues:

$z = 0$ $\quad$ $z = \pm\sqrt{2g^2 - \gamma^2}$

$\gamma < \sqrt{2}g$	all real
$\gamma > \sqrt{2}g$	(2) complex [PT-symmetry broken]

Open quantum systems

Open quantum systems: microscopic description of the environment in terms of an energy continuum

$$H=\sum_{j=1}^{n}\varepsilon_j d_j^\dagger d_j + \int_{-\pi}^{\pi} dk\, E_k c_k^\dagger c_k + \sum_{j=1}^{n}\int_{-\pi}^{\pi} dk\, V_k (c_k^\dagger d_j + d_j^\dagger c_k)$$

<u>Energy continuum</u> E_k

Well defined range $E_k \in [E_{\mathrm{th}}, \infty]$

"structured reservoir"

and density of states $\rho(E)=\dfrac{dk}{dE}$

P. Lambropoulos, G. M. Nikolopoulos, T. R. Nielsen, and S. Bay, Rep. Prog. Phys. **63**, 455 (2000).

Open quantum systems

Open quantum systems: microscopic description of the environment in terms of an energy continuum

$$H = \sum_{j=1}^{n} \varepsilon_j d_j^\dagger d_j + \int_{-\pi}^{\pi} dk\, E_k c_k^\dagger c_k + \sum_{j=1}^{n} \int_{-\pi}^{\pi} dk\, V_k (c_k^\dagger d_j + d_j^\dagger c_k)$$

<u>Energy continuum</u> E_k

Interaction between discrete system and continuum gives rise to resonance states with complex eigenvalues

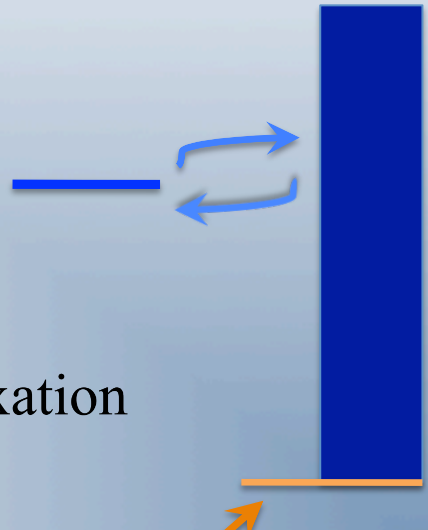
$$z_R = E_R - i\frac{\gamma}{2}$$

Origin of resonance in open quantum systems

The resonance appears in quantum systems due to the interaction of discrete states with the continuum

resonance eigenvalue: $E_R - i\frac{\Gamma}{2}$

$$P(t) = P_0 e^{-\Gamma t}$$



Examples: nuclear decay, atomic relaxation

However:
continuum threshold introduces deviations from exponential decay

L. A. Khalfin, Sov. Phys. -JETP, **6**, 1053 (1958).

So far, I have introduced two types of non-Hermitian systems

- PT-symmetric systems (explicit non-Hermiticity)

- open quantum systems (implicit non-Hermiticity)

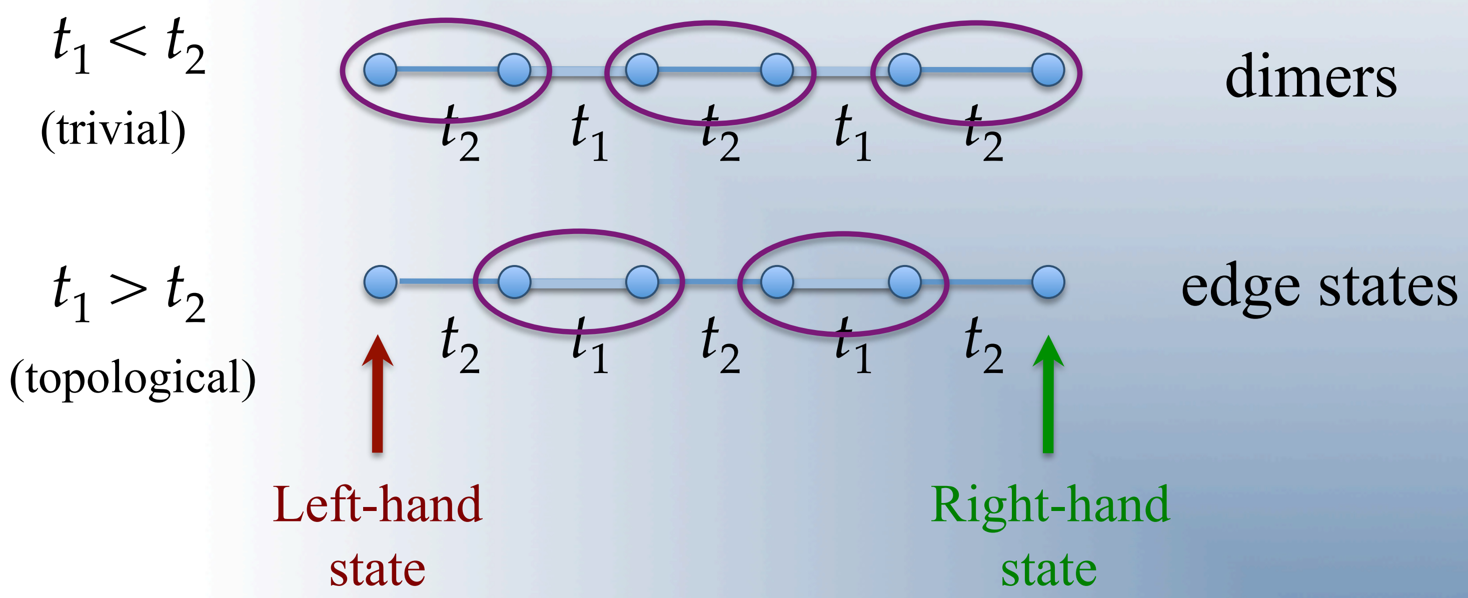
In the next step, I will combine these two in order to observe a qualitatively distinct type of PT-symmetry breaking

However, first I will need to introduce an additional concept: Protected *edge states* of a topological insulator

Su-Schrieffer-Heeger (SSH) model

Su-Schrieffer-Heeger (SSH) model

Two topological edge states can appear with $E \approx 0$



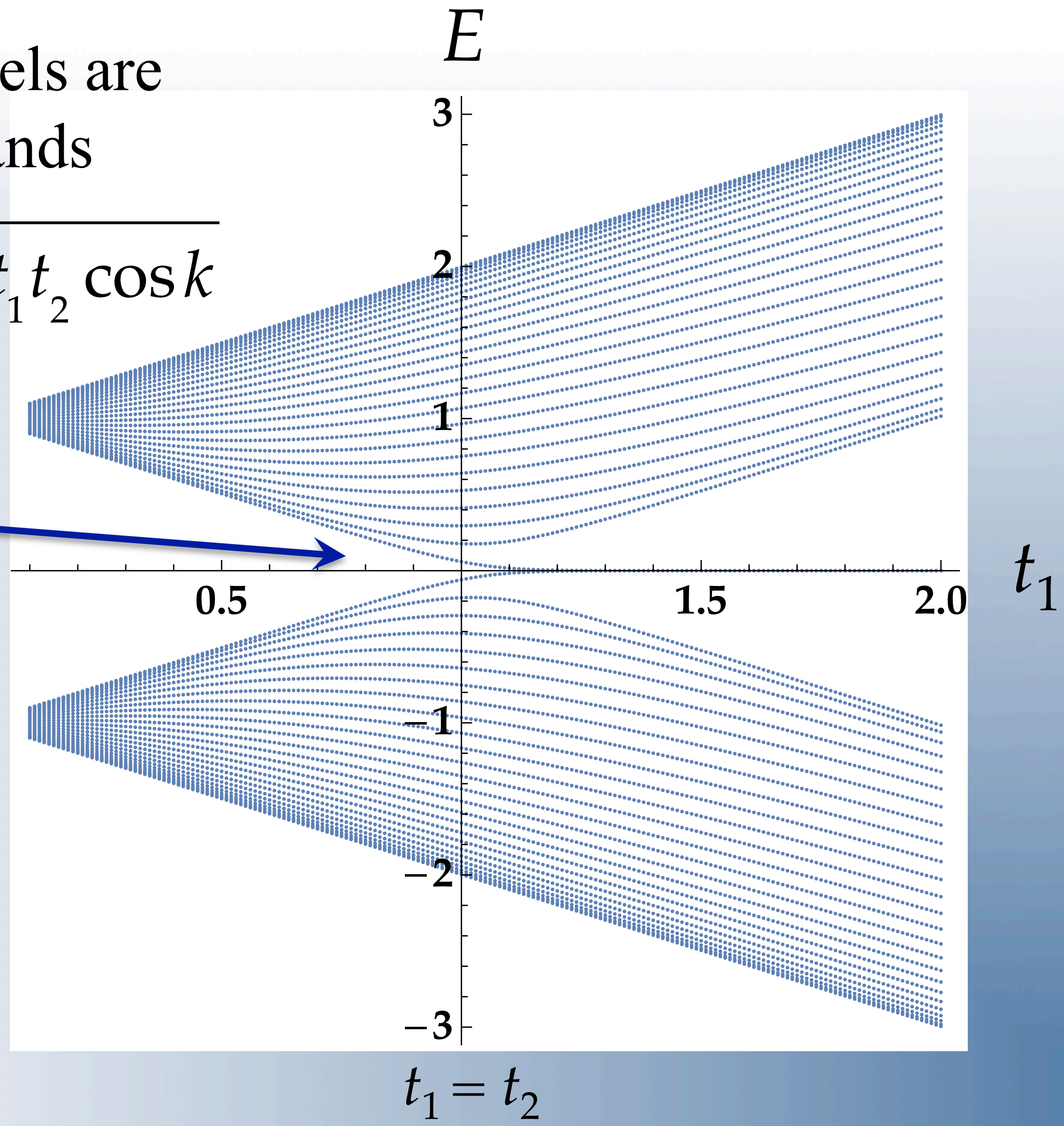
J. K. Asbóth, L. Oroszlány, and A. Pályi, Lecture Notes in Physics **919** (Springer International Publishing, Switzerland 2016).

Edge states in the SSH model: spectrum

Most SSH energy levels are organized into two bands

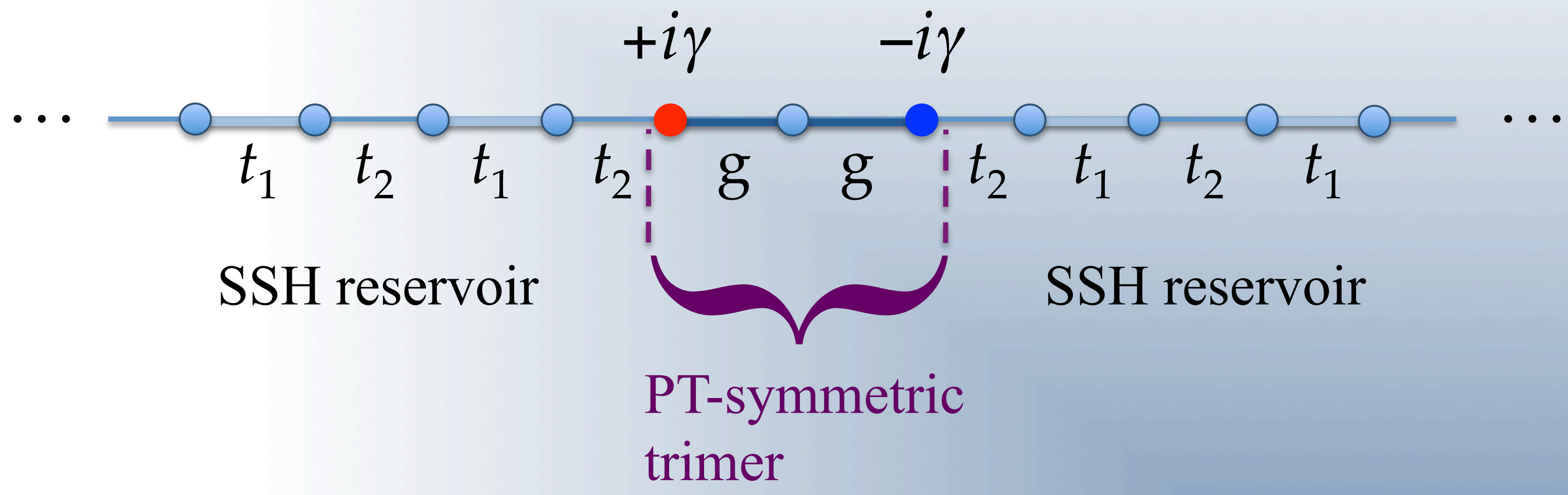
$$E_k = \pm\sqrt{t_1^2 + t_2^2 + 2t_1 t_2 \cos k}$$

edge states with $E \approx 0$ split off from SSH bands for $t_1 > t_2$



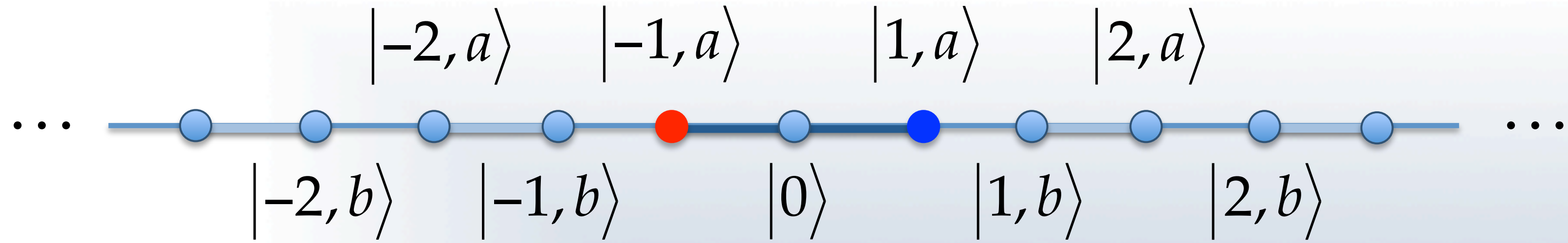
An open PT-symmetric SSH model

In this work we incorporate a PT-symmetric potential into an SSH model to study resonance and topological properties.



S. Garmon and K. Noba, Phys. Rev A **104**, 062215 (2021)

An open PT-symmetric SSH model



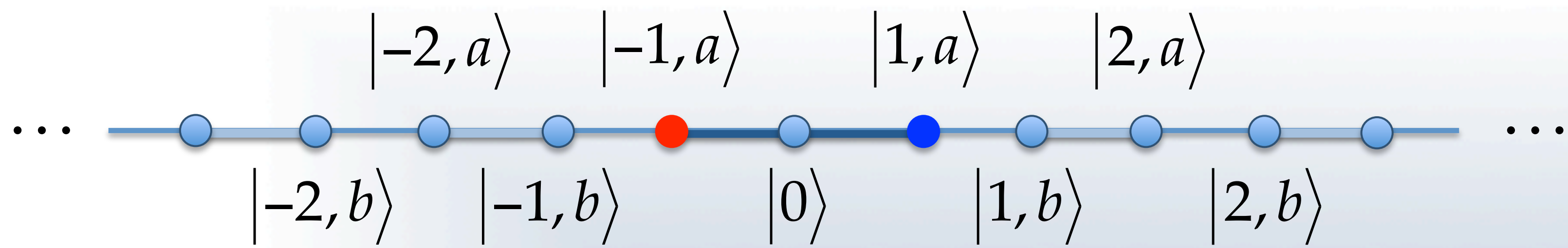
Sublattice operator:

$$\Sigma_z = P_a - P_b$$

Sublattice projectors:

$$P_a = \sum_{j=-\infty}^{-1} |j,a\rangle\langle j,a| + \sum_{j=1}^{\infty} |j,a\rangle\langle j,a| \qquad P_b = \sum_{j=-\infty}^{-1} |j,b\rangle\langle j,b| + |0\rangle\langle 0| + \sum_{j=1}^{\infty} |j,b\rangle\langle j,b|$$

An open PT-symmetric SSH model



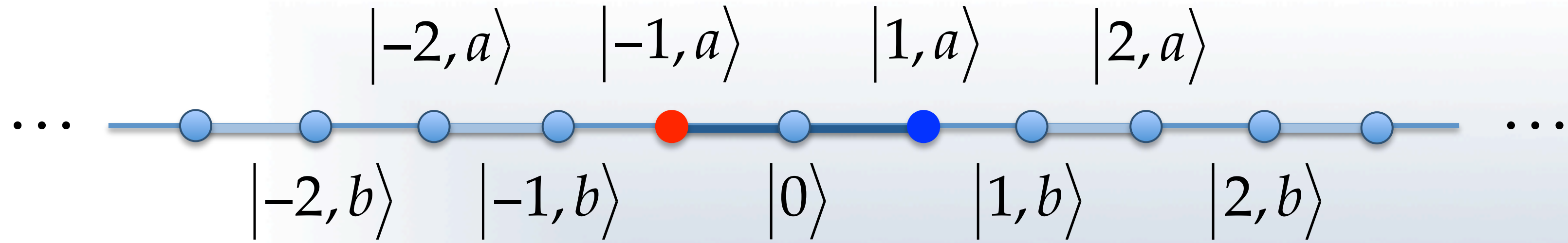
Sublattice operator:

$$\Sigma_z = P_a - P_b$$

Pseudo-anti-Hermitian symmetry:

$$\Sigma_z H \Sigma_z = -H^\dagger$$

An open PT-symmetric SSH model



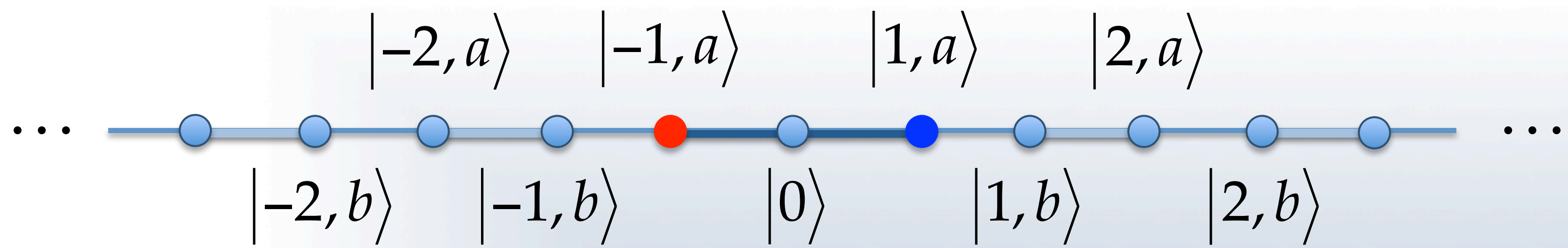
Outgoing waves (Siegert) boundary condition:

$$\begin{pmatrix} \psi_{n,a} \\ \psi_{n,b} \end{pmatrix} = e^{ikn} \begin{pmatrix} C_a \\ C_b \end{pmatrix} \qquad n > 0$$

$$\psi_{n,x} \equiv \langle \psi | n, x \rangle$$

$$\begin{pmatrix} \psi_{n,a} \\ \psi_{n,b} \end{pmatrix} = e^{-ikn} \begin{pmatrix} B_a \\ B_b \end{pmatrix} \qquad n < 0$$

An open PT-symmetric SSH model



Spectrum consists of three types of solutions

two SSH energy bands

$$E_k = \pm\sqrt{t_1^2 + t_2^2 + 2t_1 t_2 \cos k}$$

Four ordinary discrete solutions:

$$P_s(z) = \gamma^2 z^4 + \left[\gamma^4 - 2\gamma^2(t_2^2 + g^2) + t_1^4 - 2g^2 t_1^2\right] z^2$$

(double quadratic)

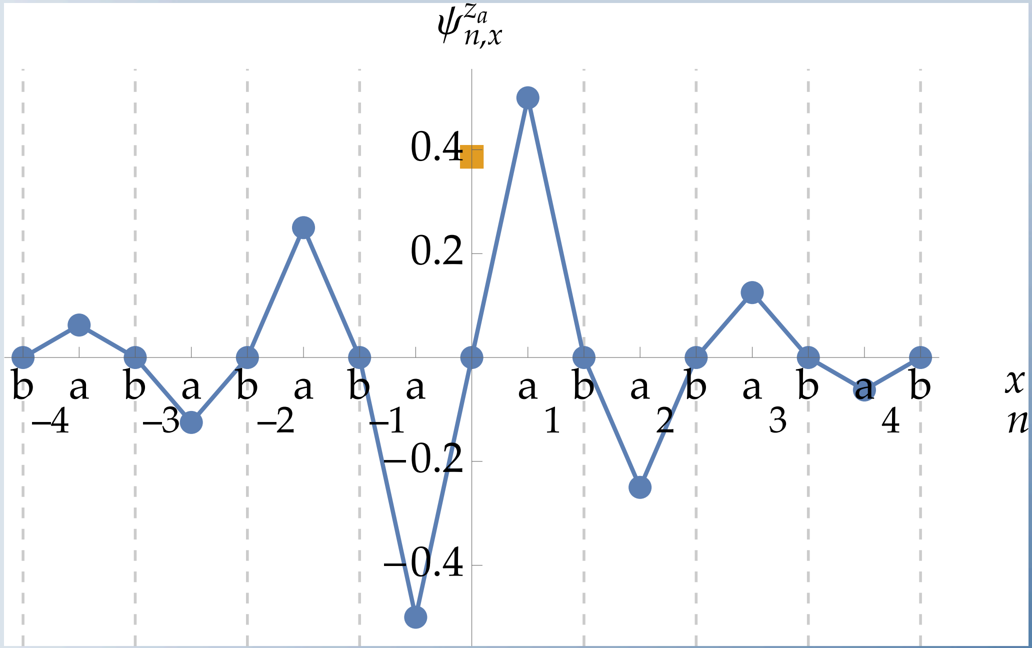
$$+ (t_1^2 - t_2^2 - 2g^2)[\gamma^2(t_1^2 - t_2^2) - 2g^2 t_1^2] = 0$$

Two zero-energy modes: $|\psi^{z_a}\rangle, \quad |\psi^{z_b}\rangle \qquad z_a = z_b = 0$

Zero-energy states

For any choice of the parameter values, there exist two (edge-like) zero-energy states.

But instead of right and left-handed modes, here we have ***localized*** and ***anti-localized*** edge-like modes



Localized condition:

$$e^{ik} = -t_2 / t_1$$

(assumption: $t_1 > t_2$)

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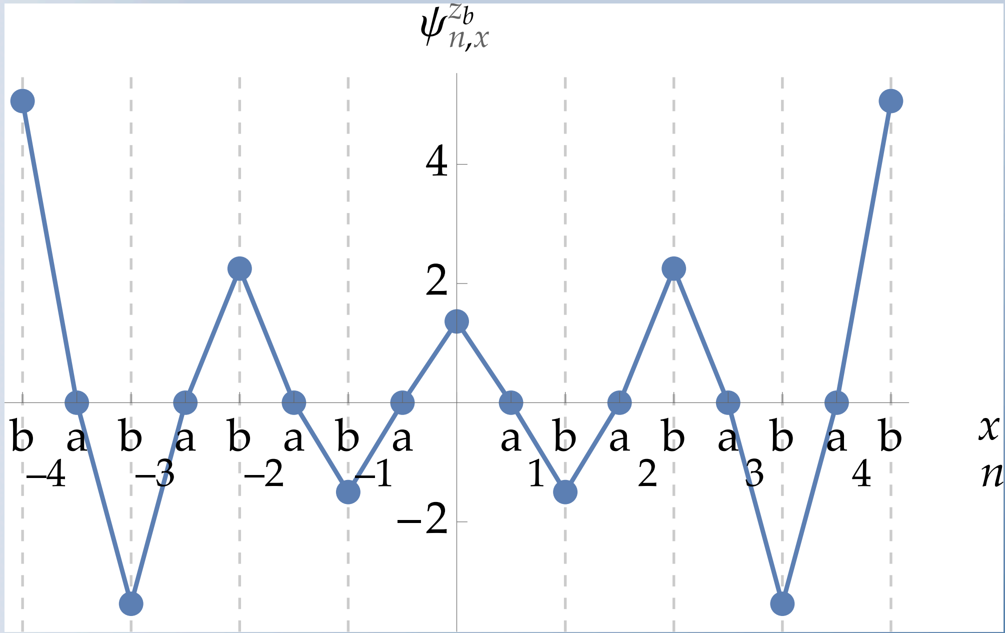
Localized condition:

$$e^{ik} = -t_2 / t_1$$

Anti-localized:

$$e^{ik} = -t_1 / t_2$$

(assumption: $t_1 > t_2$)



Zero-energy states

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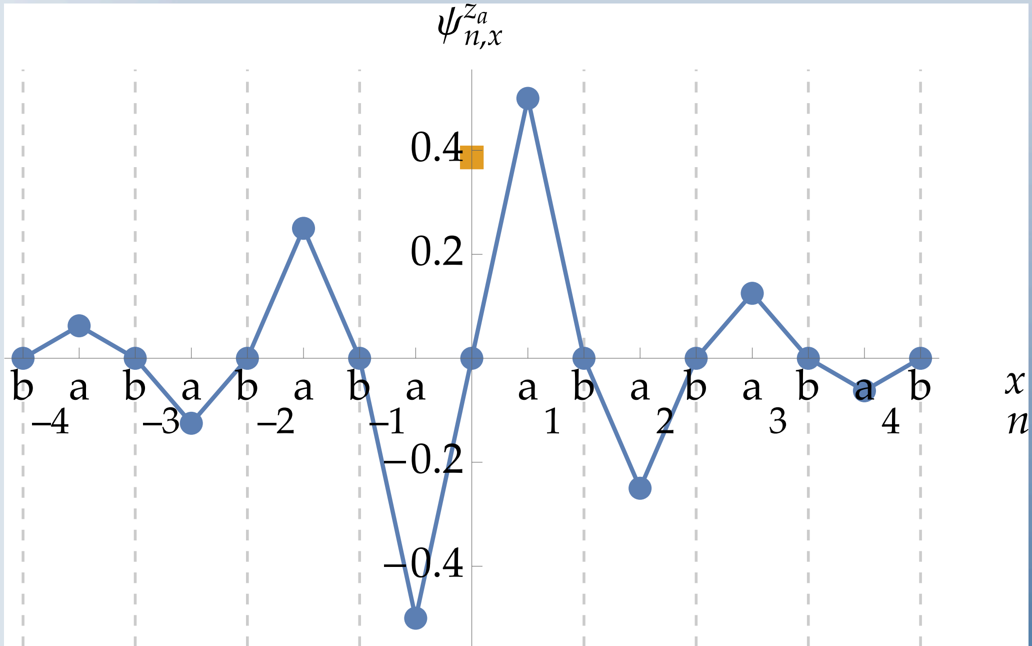


localized state has been observed in experiment…

Observation of localized zero-energy state

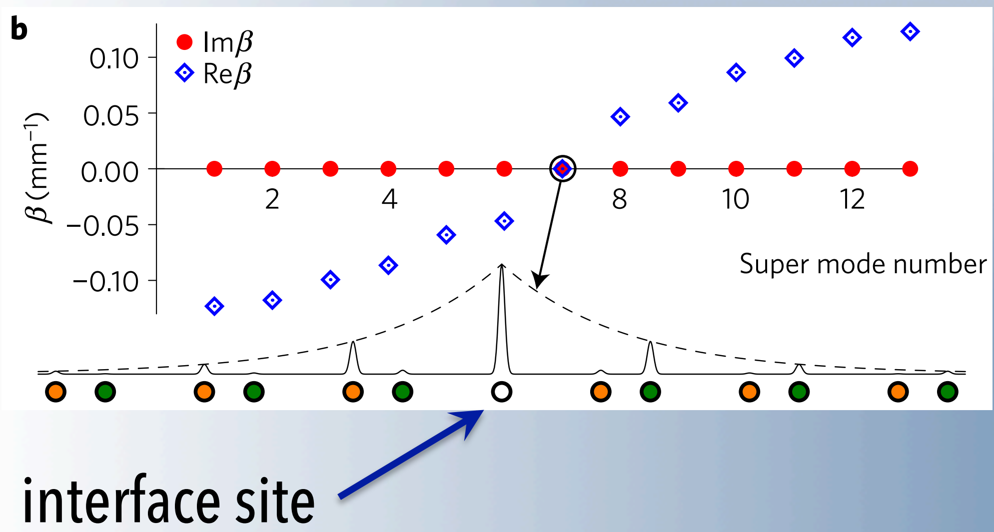
nature materials

ARTICLES

PUBLISHED ONLINE: 5 DECEMBER 2016 | DOI: 10.1038/NMAT4811

Topologically protected bound states in photonic parity–time-symmetric crystals

S. Weimann[1†], M. Kremer[1†], Y. Plotnik[2], Y. Lumer[2], S. Nolte[1], K. G. Makris[3,4], M. Segev[2], M. C. Rechtsman[5] and A. Szameit[1*]



S. Weimann, et al, Nat. Mat. **16**, 433 (2016).

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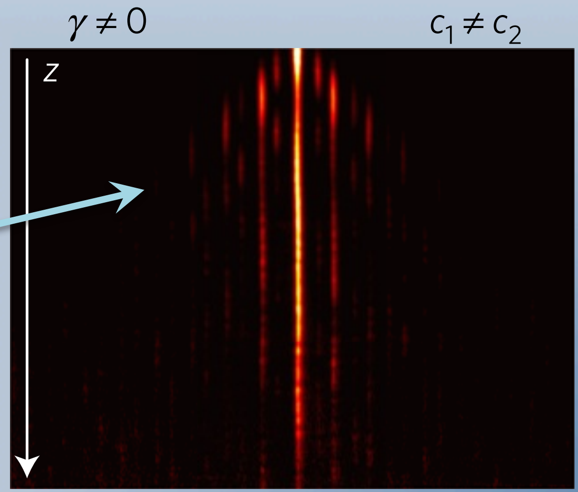
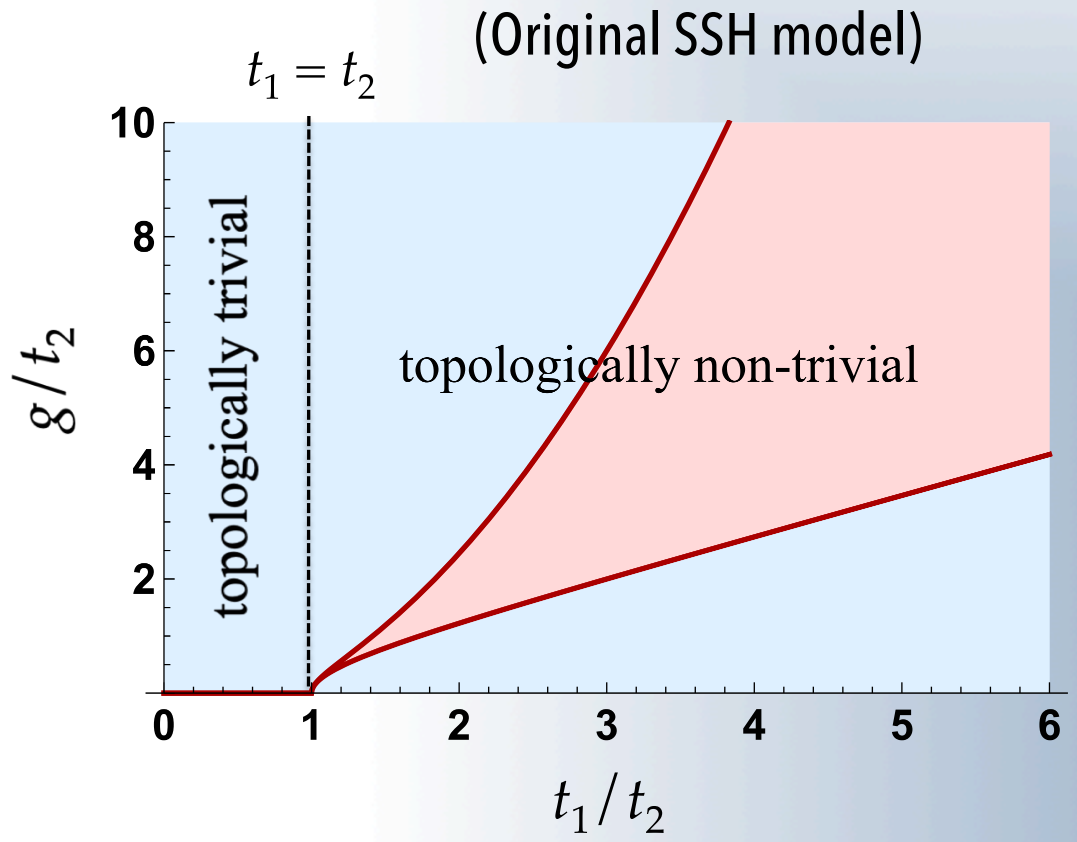
topological PT-symmetric interface state



S. Weimann, et al, Nat. Mat. **16**, 433 (2016).

Spectral phase diagram

The four polynomial solutions from $P_s(z) = 0$ can be categorized by the phase diagram:



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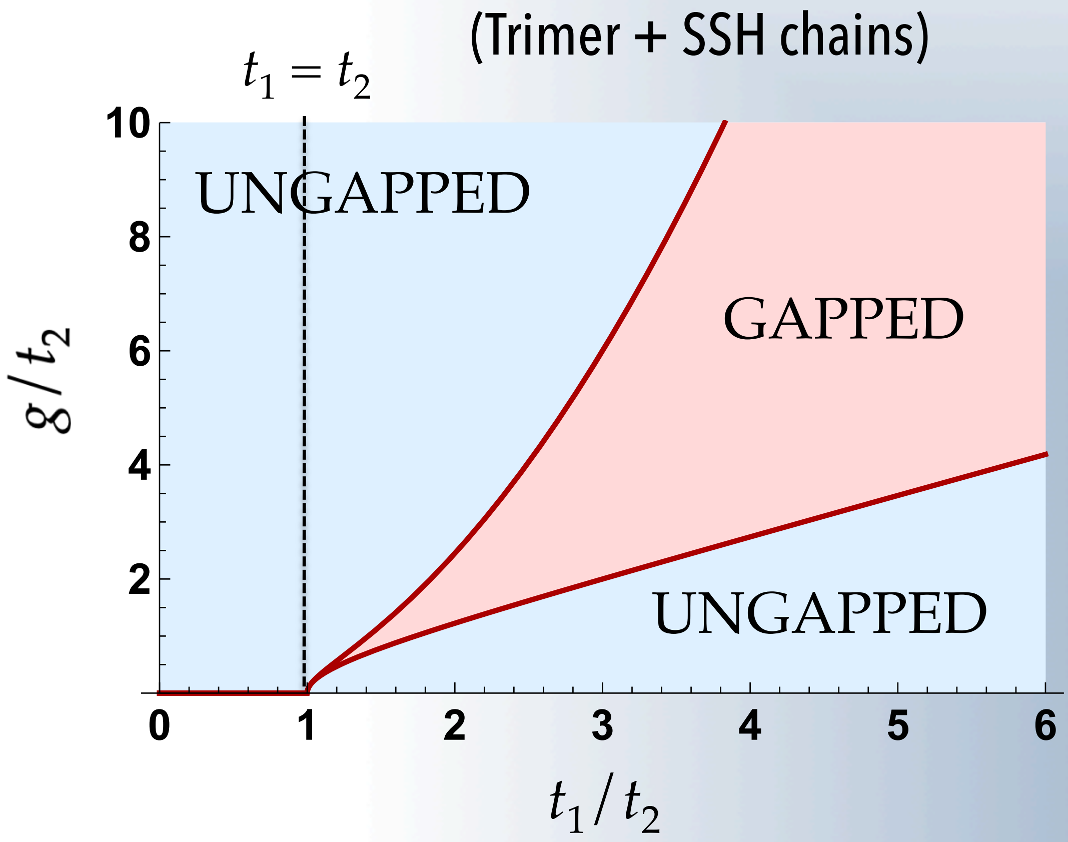
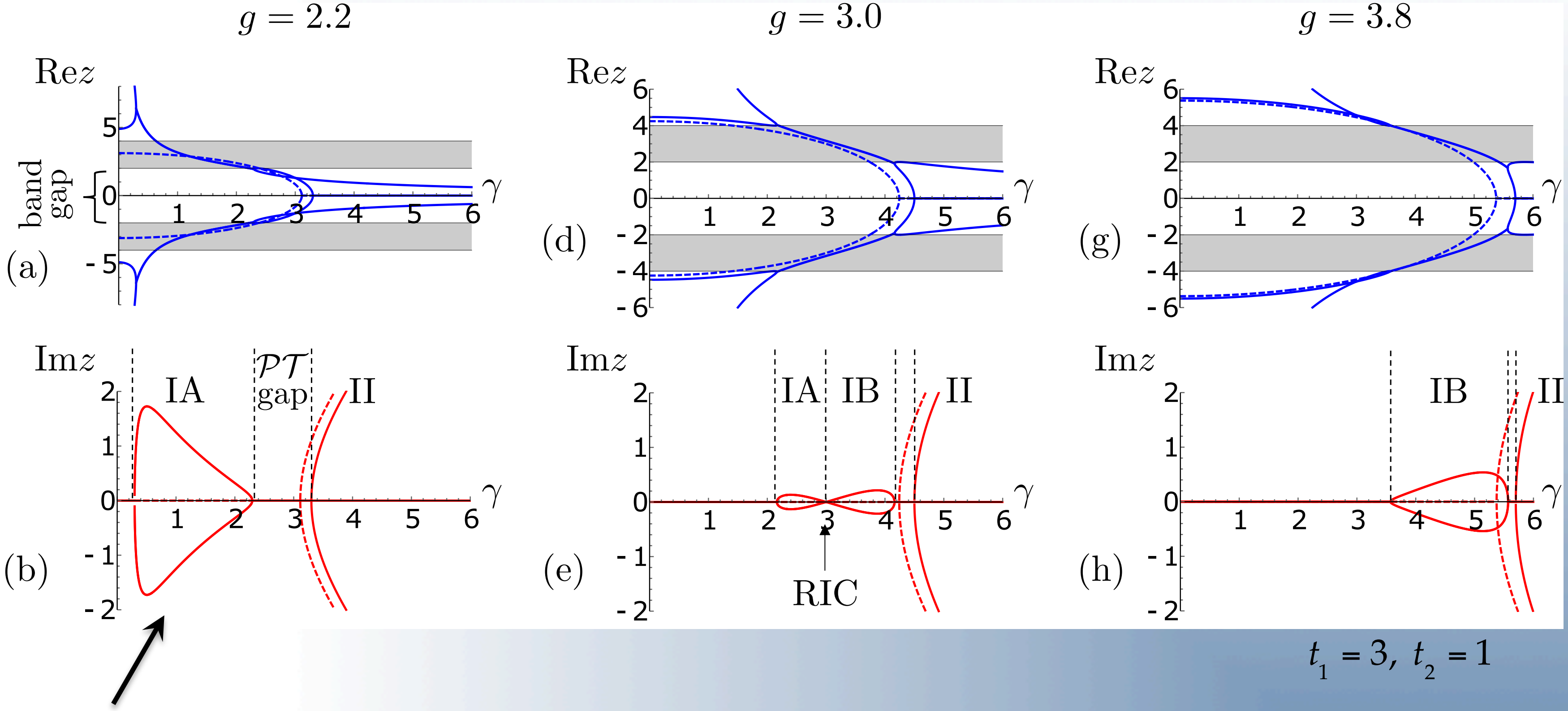


In the gapped region, we find an example of *re-entrant* PT-symmetry

Y. N. Joglekar and B. Bachi, J. Phys. A: Math. Theor. **45**, 402001 (2012).

Reservoir-assisted PT-symmetry breaking

We find there are two regions of broken PT-symmetry.



$t_1 = 3,\ t_2 = 1$

Region I – PT-symmetry is broken mainly due to the resonance interaction with the continuum.

Reservoir-assisted PT-symmetry breaking

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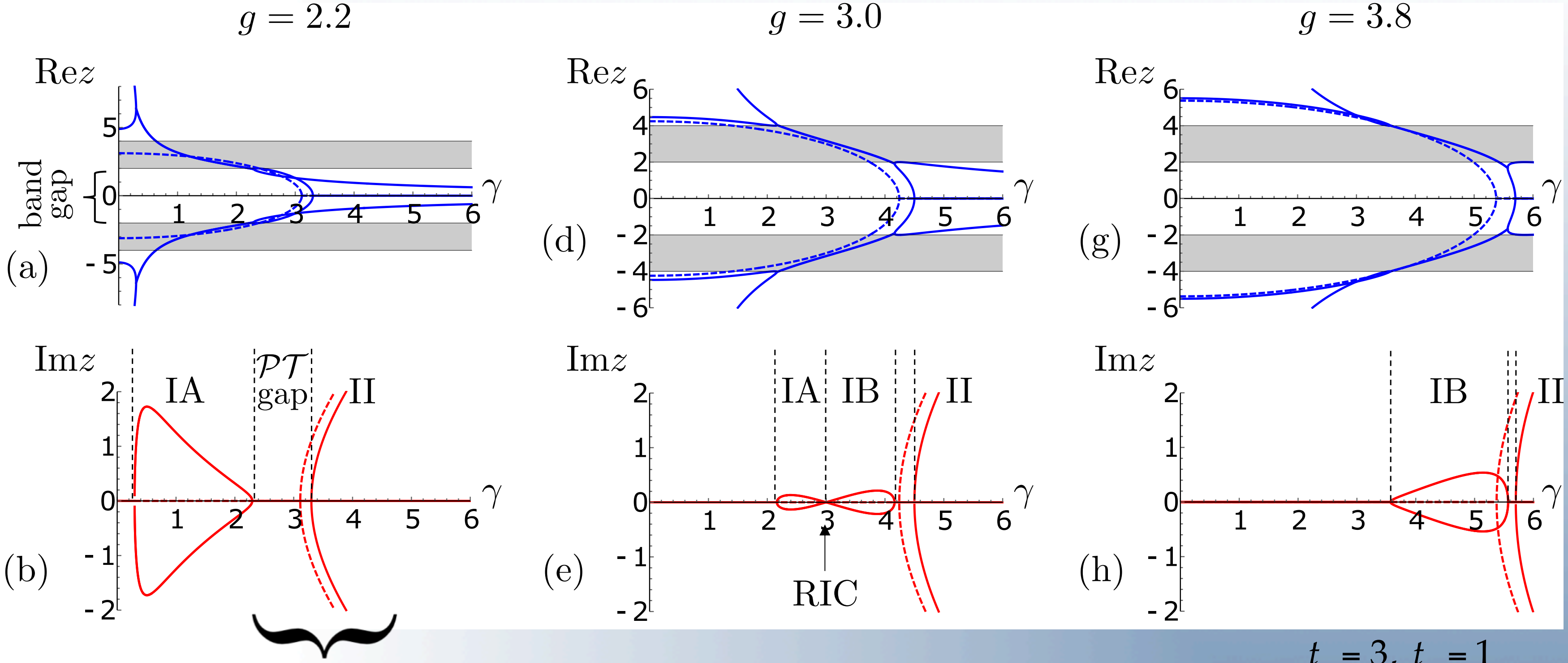


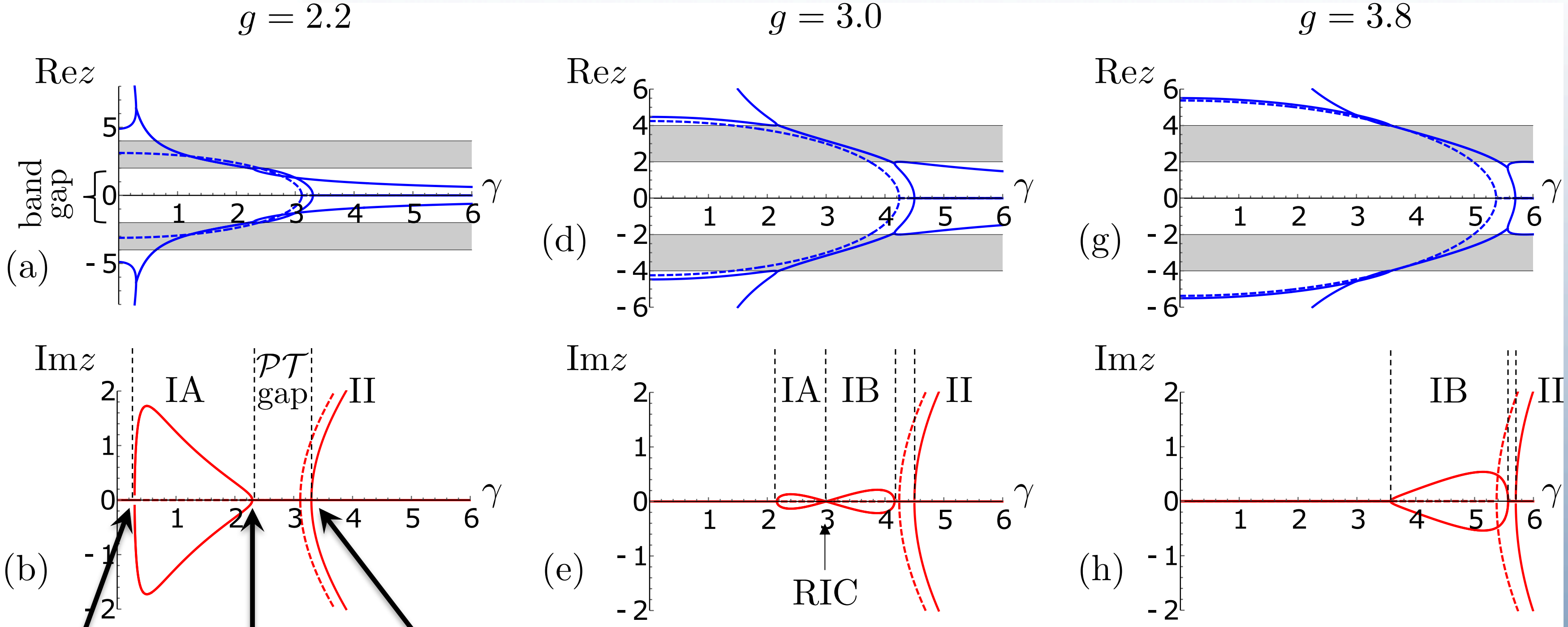
$t_1 = 3,\ t_2 = 1$

Central PT-symmetric system approximately de-couples due to localization effect

Region II – similar to broken symmetry of the decoupled PT trimer

Gapped case: exceptional points

Two broken PT-symmetry regions are delineated by <u>exceptional points</u>



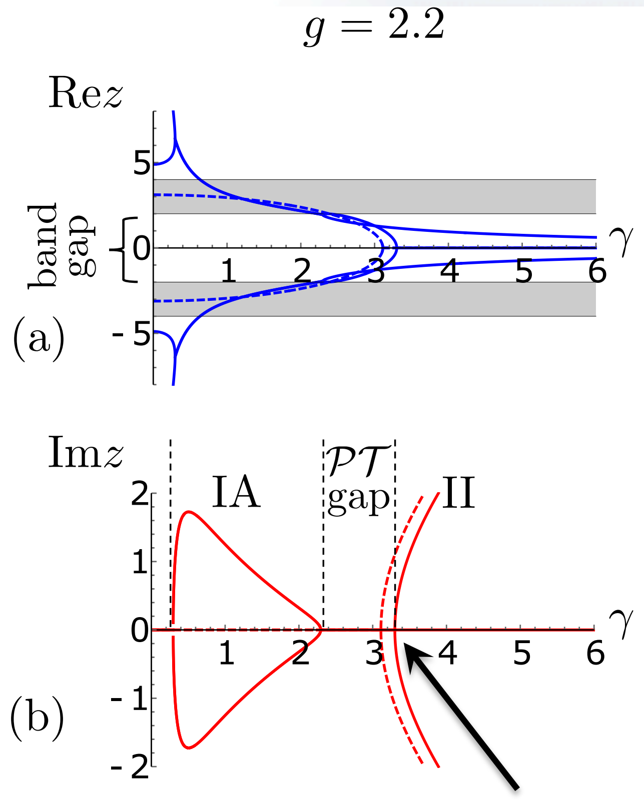
$\bar{\gamma}_{I-}$ $\bar{\gamma}_{I+}$ $\bar{\gamma}_{II} = \sqrt{2}g\dfrac{t_1}{\sqrt{t_1^2 - t_2^2}}$

$t_1 = 3,\ t_2 = 1$

$$\boxed{\bar{\gamma}_{I\mp} = \mp t_2 + \sqrt{2g^2 + t_2^2 - t_1^2}}$$

Coalesced zero-energy mode: Region II EP

Superficially, the Region II EP appears to be second-order. But it is actually third order (EP3).



$t_1 = 3,\ t_2 = 1$

$$\bar{\gamma}_{II} = \sqrt{2}g\frac{t_1}{\sqrt{t_1^2 - t_2^2}}$$

Puiseux expansion is typical of EP2

$$z_{II,\pm} = \pm\frac{1}{t_1}\sqrt{\frac{\left(t_1^2 - t_2^2\right)^3}{2g^2t_2^2 - t_1^2\left(t_1^2 - t_2^2\right)}}\left(\gamma^2 - \bar{\gamma}_{II}^2\right)^{1/2}$$

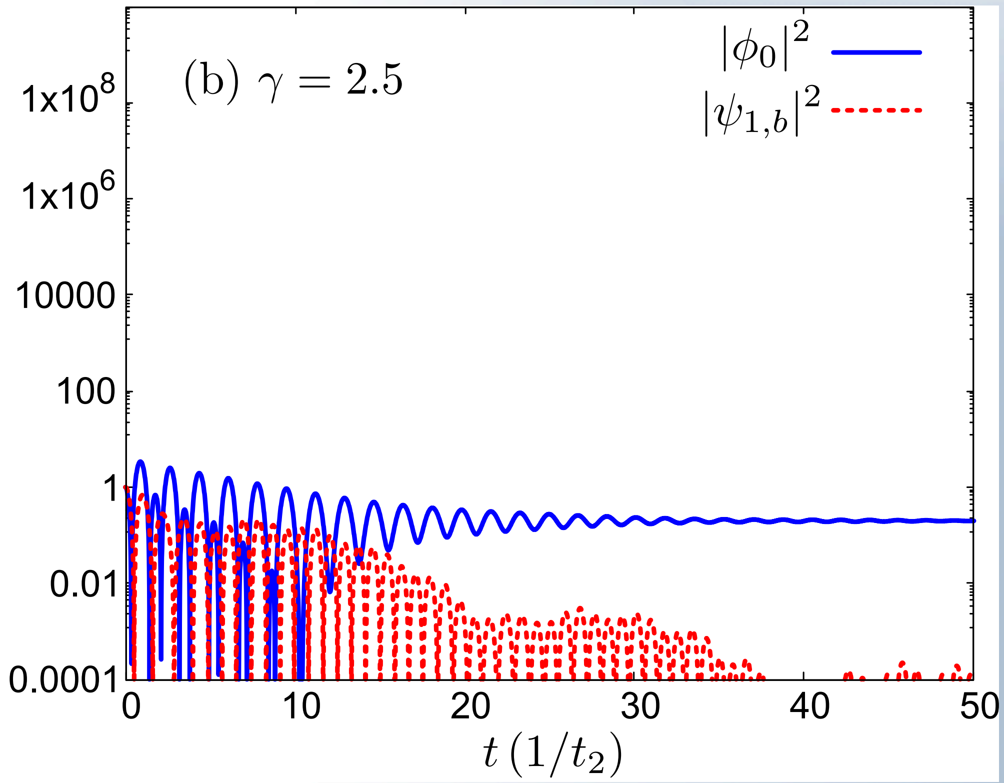
But the coalescence occurs at $z_{II} = 0$ with the condition:

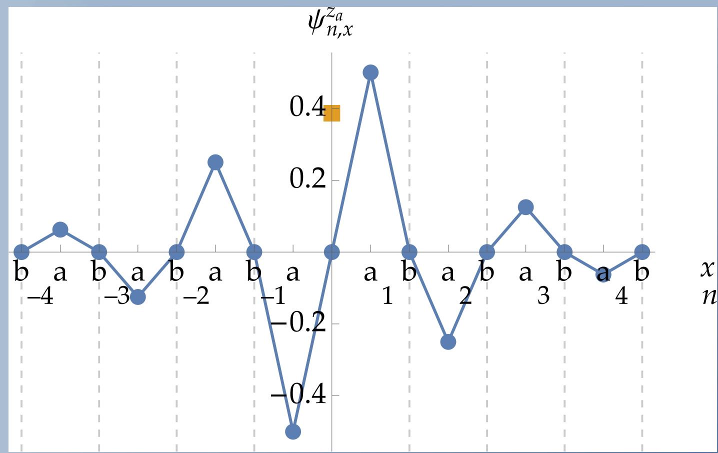
$$e^{ik} = -t_2/t_1$$

Two states are coalescing with the localized zero-energy mode

Region IA evolution: localized zero-energy state

Fractional decay occurs due to localized zero-energy state





Region IA evolution: localized zero-energy state

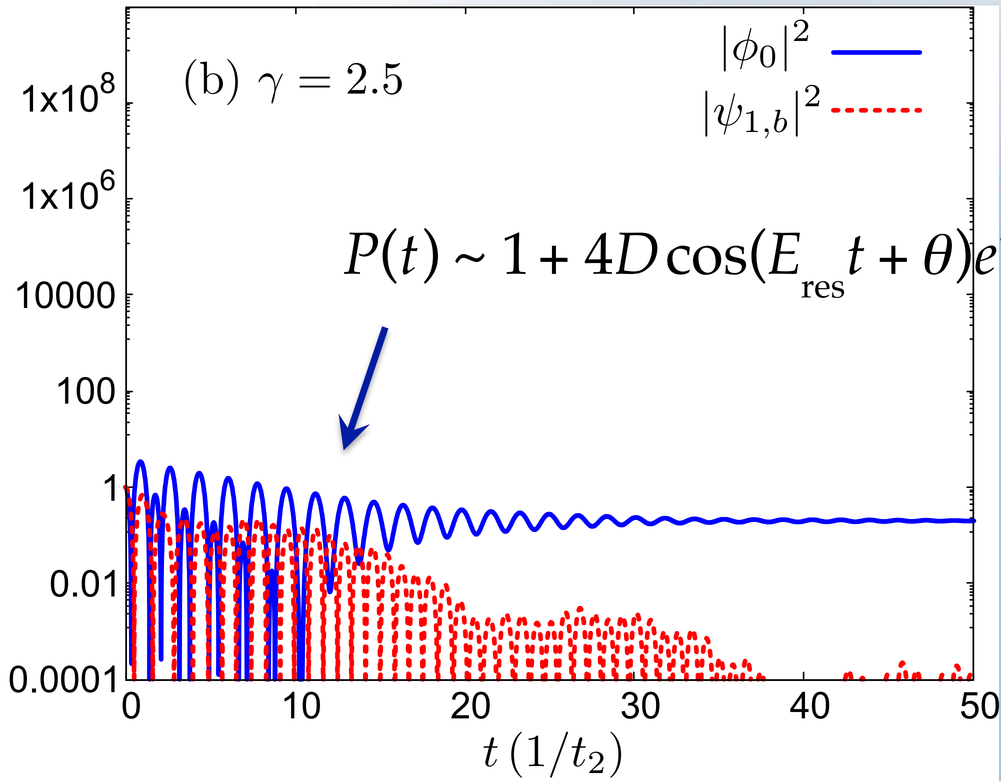
Fractional decay occurs due to localized zero-energy state

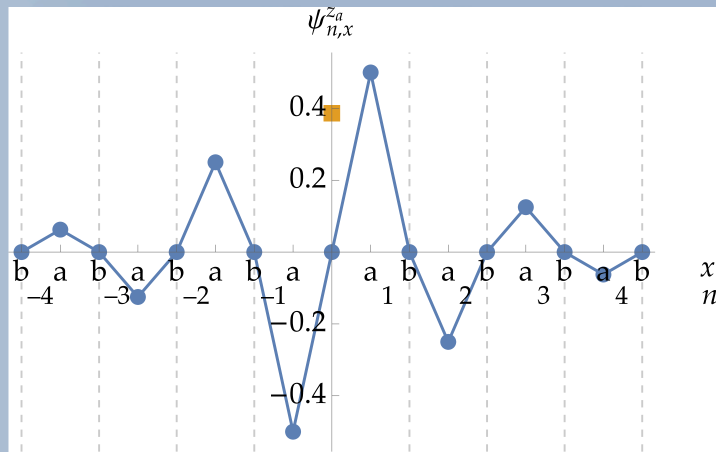
zero-energy pole: $A_{ZS,a}(t) \sim 1$

Resonance poles:

$$A_{res}(t) \sim D e^{(-iE_{\text{res}} - \gamma/2)t} + D^* e^{(iE_{\text{res}} - \gamma/2)t}$$

$$P(t) \sim 1 + 4D\cos(E_{\text{res}}t + \theta)e^{-\gamma t/2} + 4D^2\cos^2(E_{\text{res}}t + \theta)e^{-\gamma t}$$

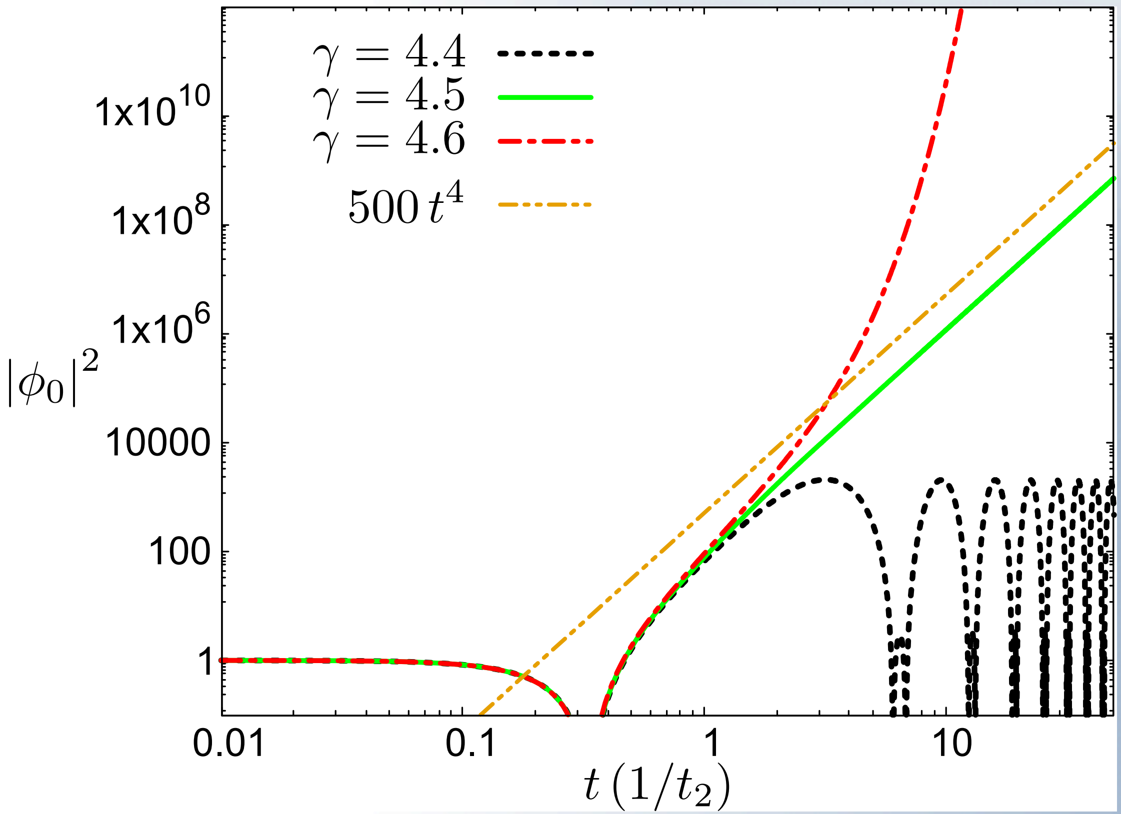




Zero-energy state EP3 dynamics

Influence of the EP3 involving the localized zero-energy state can be felt in the survival probability

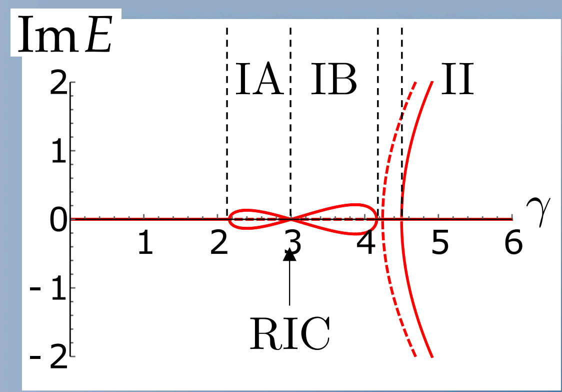
nth order pole: $A(t) \sim t^{n-1} e^{-izt}$



$P(t) \sim t^{2n-2} e^{-\gamma t}$

Since $z_{II} = 0$

$P(t) \sim t^4$ (EP3)



Conclusions

We considered a PT-symmetric system with SSH reservoirs

- Reservoir-assisted PT-symmetry breaking

- Localized zero-energy mode
  - comparable to the topological interface mode observed in a photonic lattice experiment

- Higher-order EPNs formed with localized zero-energy mode
  - Characteristic non-Markovian power law growth

$$P(t) \sim t^{2N-2}$$

S. Garmon and K. Noba, Phys. Rev A **104**, 062215 (2021)

Modification of exponential decay near EPs

Exponential decay can be modified in the vicinity of exceptional points

Consider <u>two coalescing resonance states</u>

$$A(t) = \langle q | e^{-iHt} | q \rangle = \frac{1}{2\pi i} \int_{C_E} dz \, e^{-iEt} \langle q | \frac{1}{E - H} | q \rangle$$

degenerate eigenvalues
gives a double pole

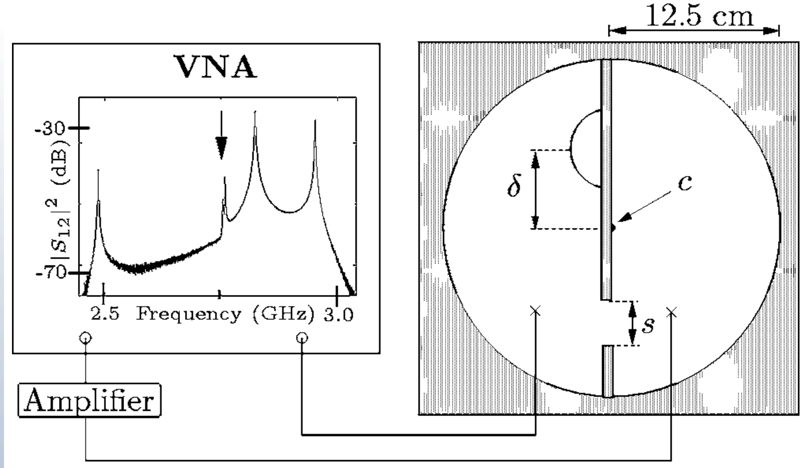
$$A(t) \sim (1 + Ct) e^{-i\bar{E}_B t - \bar{\gamma} t/2} \qquad P(t) = |A(t)|^2 \sim (1 + C_1 t + C_2 t^2) e^{-\bar{\gamma} t}$$

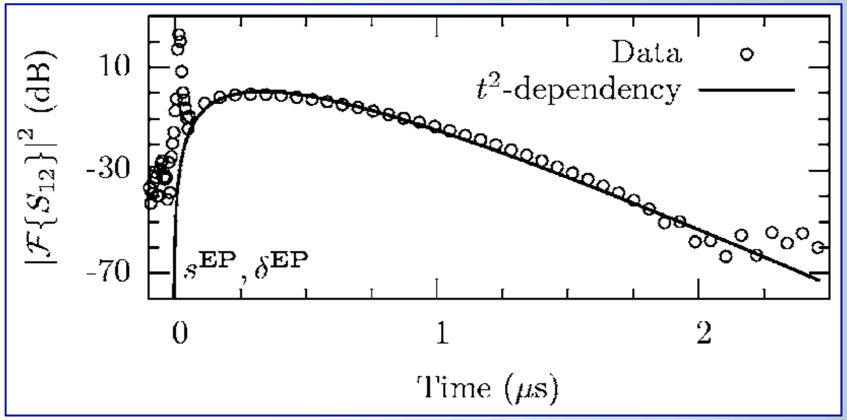
M. L. Goldberger and K. M Watson, Phys. Rev. **136**, B1472 (1964).

J. S. Bell and C. J. Goebel, Phys. Rev. **138**, B1198 (1965).

Experiment: EP2B power law-exponential decay

The power-law exponential decay has been verified in a microwave cavity experiment





$$\lim_{(s,\delta)\to\mathbf{EP}} |\widetilde{\mathbb{G}}_{12}(t)|^2 \approx \frac{\sigma^2}{\mathcal{E}_+\mathcal{E}_-} t^2 e^{-\bar{\gamma}t}$$

B. Dietz, *et al*, Phys. Rev. E **75**, 027201 (2007).

S. Bittner, *et al*, Phys. Rev. E **89**, 032909 (2014).