Rigged Hilbert Space formulation for Non-Hermite System with Positive Definite Metric

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Abstract

A rigged Hilbert space (RHS), also called Gel'fand's triplet, is the space composed of the triplet of topological vector spaces,

$$\Phi \subset \mathcal{H} \subset \Phi', \tag{1}$$

where $\mathcal{H}$ is a complex Hilbert space, Φ is a dense subspace of $\mathcal{H}$ that equips a nuclear topology, and Φ' is a dual space of Φ. RHS was first introduced mathematically by I. M. Gel'fand and his collaborators as a means of associating distribution theory with Hilbert space theory [1, 2]. Nowadays, this space is recognized as one of the underlying spaces for describing quantum mechanics [3, 4, 5]. In particular, the nuclear spectral theorem of RHS shows the spectral expansions of the bra and ket vectors through the eigenvectors of the observables, and the Dirac's bra-ket formalism can be formulated from the spectral expansions. However, the mathematical studies of modern quantum mechanics, such as non-Hermitian systems, using RHS are still insufficient.

In my talk, focusing on a non-Hermitian system with a positive definite metric, its RHS formulation is introduced [6]. First, I will review the definition, nuclear spectral theorem, and how the Dirac's bra-ket formalism is formulated, for RHS. Then, a RHS associated with the positive definite metric is established. Based on the obtained space, the spectral expansion for quasi-Hermitian and bra-ket formalism for the non-Hermite system are shown. Finally, as the application to a physical model, the Swanson model is focused on [7].

References

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